THE COST OF EQUITY UNDER THE AUSTRALIAN DIVIDEND IMPUTATION TAX SYSTEM

by

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Abstract:

This paper derives a Capital Asset Pricing Model ("CAPM") in the context of the Australian dividend imputation tax system. The effect of dividend imputation on the estimated risk premium is discussed. The structure and implications of the derived CAPM are also examined.

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1. Introduction

The valuation of projects and companies by discounted cash flow techniques is a widely accepted valuation methodology. Discounted cash flow analysis requires both the estimation of the likely cash flows and the determination of an appropriate discount rate. Conceptually at least, cash flows to equity holders can be considered (i) before all taxes, or (ii) after corporate tax but before investor-level taxes, or (iii) net of all taxes. However, equity holders can only claim that portion of the cash flow that remains after the payment of both corporate tax and investor-level taxes. Thus, if the cash flows being discounted are after corporate tax but before investor-level taxes, the discount rate must, in general, take account of investor-level tax payments.

Prior to the introduction of the dividend imputation tax system in 1987, a standard approach to the valuation of firms consisted of the following two steps. First a cash flow stream accruing to shareholders was estimated. This cash flow stream was after corporate tax but before investor-level taxes. Second, these cash flows were then discounted at a cost of capital that was on a before investor-level tax basis. Frequently, the cost of capital was estimated using the "classical" CAPM.[1] This approach did not explicitly consider investor-level taxes and was valid only if it was assumed that investor-level taxes were consistent in their treatment of income and capital gains.[2] While this assumption was not strictly valid, the resulting errors seem likely to be small when compared with making the same assumption in the current dividend imputation tax system. Under dividend imputation, dividends can carry imputation credits, or tax credits, to domestic end investors for Australian corporate tax paid. For Australian tax-paying investors, this results in the investor-level tax rate on fully franked dividend income being, in general, substantially less than the investor-level tax rate on capital gains.[3] Thus the traditional assumption that investor-level taxes are consistent in their treatment of income and capital gains appears invalid in the current dividend imputation tax system. Another consequence of the dividend imputation tax system is that a cash flow carrying imputation credits has a substantially higher value to Australian tax-paying investors than a cash flow carrying no imputation credits.[4] However, the traditionally accepted valuation method ignores the imputation credits associated with the cash flow and essentially ascribes zero value to the imputation credits. Hence, for these reasons, a new model is

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1. Although the valuation of companies and projects typically involves multi-period cash flows it is noted that the CAPM derived in this paper is only applicable to a single, end-of-period cash flow. The extension of the (single-period) CAPM to the multi-period case is not without difficulties. The problems and further references are discussed in T.E. Copeland & J.F. Weston, *Financial Theory and Corporate Policy*, 3rd ed., Addison-Wesley Publishing Company, Reading, Massachusetts, 1988, pp. 401–411.
2. R.A. Brealey & S.C. Myers, *Principles of Corporate Finance*, 4th ed., McGraw-Hill Book Company, New York, 1991, p. 428.
3. Table 1 below illustrates how, for an Australian investor with a tax rate of 15%, the net investor-level tax rate on fully franked dividend income is –27% which is obviously well below their tax rate on capital gains. More generally, the delay of capital gains tax until the gain is realised, and the fact that only real gains are taxed, means that, effectively, capital gains are taxed at less than the investor's nominal tax rate.
4. Table 1 below illustrates that a $0.67 franked dividend is worth $0.53 after all taxes to an Australian taxpayer with a marginal tax rate of 47%. For such a taxpayer, a $0.67 unfranked dividend is worth only $0.36 after all taxes.

needed to meet the changed circumstances. This paper develops such a model from the foundation of portfolio theory.

The effects of investor-level taxes on the CAPM were examined by Brennan[5] in the context of the "classical" United States tax system. In particular, Brennan considered the asymmetric tax treatment of dividend income and capital gains. Ashton[6] derived a CAPM for the British imputation tax system but assumed that a company's dividend policy did not affect the value of the firm. This paper extends the analysis undertaken by Brennan and Ashton by modelling specific features of the Australian dividend imputation tax system, and by modelling Australian corporate tax payments. Whilst this paper follows the general methodology employed by Ashton, the conclusions do not depend on the dividend irrelevancy assumption.

This paper derives a CAPM consistent with a cash flow measured after corporate tax but before investor-level taxes in the context of the Australian dividend imputation tax system. It is assumed that the investor-level tax rates on capital gains and losses and on income not carrying imputation credits (interest, unfranked dividends, etc.) are equivalent.[7] The model allows for the possibility that some investors may not be able to fully utilise imputation credits distributed with franked dividends, and that undistributed imputation credits retained in a firm's franking account may increase the firm's equity value.

2. The Australian Imputation Tax System

The key features of the Australian imputation tax system can be seen from Table 1.[8] At a corporate tax rate of 33%, each \$1 of (Australian sourced) taxable corporate income enables \$0.67 of "franked" dividend income to be paid. This dividend carries (to Australian taxpayers) an additional amount of imputed taxable income of \$0.33 (equal to the Australian corporate tax paid). Also carried is a tax credit of the same amount, so that the net tax payable by the investor depends upon the investor's tax rate.

Hence for Australian tax-paying investors, the tax paid on grossed-up dividend income is a function of their personal tax rates only, with Australian corporate tax essentially becoming a withholding tax. Under the previous "classical" system, tax was paid at both

5. M.J. Brennan, "Taxes, Market Valuation and Corporation Financial Policy", *National Tax Journal*, December 1970, pp. 417–427.
6. D.J. Ashton, "The Cost of Capital and the Imputation Tax System", *Journal of Business Finance and Accounting*, 16(1) Spring 1989, pp. 75–88.
7. For Australian investors the effective capital gains tax rate is less than the income tax rate for two reasons. First, capital gains are only taxed as realised and not on an accruals basis. Second, for assets held longer than 12 months, capital gains (but not losses) are only taxed on their real component (i.e. on the nominal gain less inflation over the period as measured by quarterly movements in the Australian Consumer Price Index). Note that the tax rate on capital gains equals the tax rate on capital losses only if the time period being considered is less than 12 months.
8. The Australian dividend imputation tax system is more fully explained in G. Peirson, R. Bird, R. Brown & P. Howard, *Business Finance*, 5th ed., McGraw-Hill Book Company, Sydney, 1990, pp.455–456, and D. Hamson & P. Ziegler, "The Impact of Dividend Imputation on Firms' Financial Decisions", *Accounting and Finance*, November 1990, pp. 29–53.

TABLE 1

		End Investor	
	Individual at maximum tax rate*	Superannuation Fund*	Offshore Investor**
Income tax rate	47%	15%	x%
	$	$	$
Before tax Australian corporate income	100	100	100
Australian corporate tax	(33)	(33)	(33)
After corporate tax income	67	67	67
End investor			
Dividend received (100% payout)	67	67	67
Imputation credit	33	33	0
Grossed-up dividend	100	100	67
Tax on dividend	(47)	(15)	(67x)
Franking rebate	33	33	0
Net investor-level tax paid	(14)	18	(67x)
Net total tax paid	(47)	(15)	(67x + 33)
Income after all taxes	53	85	67(1–x)

(*) Assumes end-investors can fully utilise imputation credits.
(**) This analysis ignores dividend withholding tax effects.

the corporate level, on before corporate tax income, and at the investor-level on dividends received. Dividend imputation can remove this feature of "double taxation", at least for resident taxpayers.

The Australian dividend imputation tax system still leaves an offshore investor liable to double taxation on corporate income. Hence, relative to domestic investors, offshore investors are disadvantaged,[9] assuming they cannot "sell" or otherwise capture the value associated with the imputation credits. The Australian government has moved to try to prevent the transfer of imputation credits between investors.

The payment of Australian corporate tax will disadvantage Australian tax-paying investors to the extent that there is a delay between the payment of corporate tax and the "refund" or benefit obtained by the end-investors. Accordingly, value can still be gained, at the margin, if corporate tax is delayed or minimised. Additionally, the deferral or minimisation of

9. Note that offshore investors are exempt from dividend withholding tax if the dividends are fully franked. The value of this exemption depends on whether a tax credit in their home country could have been obtained for the withholding tax that would otherwise have been paid, and whether this tax credit would have value to the offshore investor.

Australian corporate tax will obviously benefit those investors that cannot fully utilise their imputation credits.

3. Assumptions and Model Overview

The derivation of a single period CAPM, in an economy with capital gains tax, income tax and a dividend imputation tax system, requires the usual assumptions[10] which include the existence of risk-averse investors who may borrow and lend at the risk-free rate, have homogenous expectations about asset returns and are price-takers. Additionally, asset markets are assumed to be frictionless and information is assumed costless and simultaneously available to all investors.

The following additional assumptions are made:

- dividends are paid at the end of the period, and both the cash amount of the dividend and the level of franking is known with certainty at the beginning of the period;[11]
- corporate debt carries a risk-free interest rate;
- investor-level taxes, consisting of both capital gains tax and income tax, are payable at the end of the period;[12]
- the investor-level tax rates on capital gains and losses and on income are assumed to be equivalent;
- all taxes and tax credits are respectively payable and available at the end of the period;
- imputation credits distributed with franked dividends may not be able to be fully utilised, i.e. they may not be able to be rebated in full in calculating the investor's tax charge; and
- for all firms in the market, each dollar of imputation credits retained by a firm will increase that firm's equity value by a constant amount of $\$\theta^r_m$.[13] For example, if \$1.00 of imputation credits were acquired by the firm, for no consideration, and it had been assumed that θ^r_m equals 0.25 then this implies that the equity value of the firm would increase by \$0.25.

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10. T.E. Copeland & J.F. Weston, *ibid.*, p. 194.
11. Both Ashton and Brennan use this approach. It would be more realistic to assume that both the amount of the dividend and the level of franking are functions of the uncertain end-of-period cash flow. However, implementation of this approach would increase considerably the complexity of the model.
12. The payment of capital gains tax as capital profits are realised cannot be explicitly modelled within the framework of a single-period CAPM. If the single-period CAPM is extended to the multi-period case then intuitively this assumption is consistent with capital gains tax being paid on an accruals basis.
13. The symbol θ is used throughout to represent a "utilisation factor". Thus θ^r_m applies to the market value (subscript m) of retained imputation credits (superscript r). Subsequently, the superscript d is used to represent distributed imputation credits and the subscripts i and j are used to represent investor i and firm j respectively.
    Arguably θ^r_m could be firm-specific (i.e. θ^r_j) rather than market-specific and hence constant for all firms in the market. The assumption used in this paper, however, simplifies the mathematics.

The last assumption allows for the possibility that the market places a positive value on (undistributed) franking account balances – reflecting the ability this provides for the future distribution of franked dividends. However, the introduction of this assumption is not without its difficulties. In the context of a single-period model, it is usual to assume that the end-of-period wealth is distributed to shareholders. The distribution of retained imputation credits could be modelled by assuming that (i) the undistributed imputation credits are worthless (i.e. $\theta^r_m = 0$) and hence don't have to be distributed, or (ii) the undistributed imputation credits are distributed immediately to shareholders, even when a firm's dividend policy is such that not all of the imputation credits are, or can be, distributed, or (iii) the undistributed imputation credits are capitalised into the equity value of the firm and this increased value is "distributed" to shareholders. Intuitively, this increased value of equity, caused by the retention of imputation credits, is distributed to shareholders in a multi-period case by the future distribution of imputation credits, the value of which will depend upon the ability of the investors to utilise the imputation credits in the future.[14] Note that the increased equity value of the firm will benefit all shareholders equally, despite individual investors valuing the expected distribution of retained imputation credits differently. This is in contrast to the value of distributed imputation credits which varies across individual investors.

Approach (iii) is arguably the most realistic and has been used in this paper. For example, if $\theta^r_m > 0$, it allows an offshore investor (for whom distributed imputation credits may well be worthless) to benefit from retained imputation credits to the extent that they cause an increase in the share price.

The derivation of the CAPM proceeds as follows. First, a representative investor is considered and an optimality condition is found which applies to all investors. This condition is given by equation (4.8). Next it is assumed in Section 5 that all investors have optimised their portfolios and that the securities market is in equilibrium. This results in equation (5.1) which shows the market value of equity of a representative firm. After introducing some definitions for expected returns on the jth firm and the market (equations (6.1) and (6.2)), it is then possible to derive equation (6.4), which is a CAPM relevant to today's Australian tax system. This equation can be related directly to the "classical" CAPM. Finally, by defining the market return to include imputation credits (equation (7.1)), the CAPM, adjusted for Australian tax conditions, is restated in equation (7.2).

4. Investor's Optimal Portfolio

The derivation of the CAPM begins by obtaining an expression that must be satisfied if a representative (ith) investor's portfolio of securities is to be optimised. It is assumed that

14. An alternative assumption is that the increased value of equity is realised by the "sale" of the undistributed imputation credits. Although, as mentioned in Section 2, the Government has moved to try to prevent the transfer of imputation credits between investors. This approach is, however, consistent with θ^r_m being market-specific (i.e. constant for the Australian market) as the sale price would, of necessity, be determined by the market.

the portfolio of securities can be optimised by maximising the (ith) investor's utility function defined over the mean and variance of the expected portfolio value. Naturally, these measures are after all taxes as it is assumed that investors derive utility only from after-tax cash flows.

Three different classes of securities represent the complete set of securities available in the economy. They are: risk-free government debt; corporate debt (also assumed risk-free); and risky equity of the jth firm.

The first task is to determine, for each type of security, the expected value of the cash flow to the investor, after all taxes. For the debt securities this is simply the principal repayment plus interest earned, at the risk-free rate, less investor-level tax payments on the interest earned. For the risky equity cash flow, a general expression is constructed as follows. Beginning with a firm's cash flow before interest and (Australian) tax, $\mathbf{X}_j$,[15] Australian corporate tax appropriate to the jth firm is determined by taking into account the nominal corporate tax rate (T), and deductions such as: interest payments on borrowings (L_j) with interest rate (r_f), and other "general" deductions (P_j) which have been grouped together. P_j can be interpreted as the amount deducted from cash flows in calculating Australian taxable income because of factors such as depreciation, carried-forward tax losses, foreign tax payments and any cash inflow of loan funds, but does not include any tax deductibility for interest. It is essentially an adjustment to allow for the case where the effective Australian corporate tax rate on before (Australian) tax cash flows is not the nominal corporate tax rate, T. An after corporate tax and debt repayments cash flow is therefore modelled by; a cash flow before interest and tax, less tax paid after taking into account tax "savings" on deductions and interest paid, and less principal and interest paid. Using the notation defined above, the cash flow available to shareholders after corporate tax and debt repayment is $\mathbf{X}_j - (\mathbf{X}_j - P_j - r_f L_j)\,T - L_j(1 + r_f)$.

This cash flow is then subject to tax in the hands of the investor. Tax is payable on dividends received and capital gains and, as previously discussed, both income and capital gains are assumed to be taxed at the same rate (t_i). However, the amount of tax payable by the investor on dividends received depends on whether the investor is classified for tax purposes as an offshore investor or as an Australian investor. Examination of Table 1 shows that the calculation of the after-tax cash flow to these two types of investor is distinctly different. For an offshore investor, the after investor-level tax cash flow resulting from the receipt of a dividend equals:

$$s_{ij}d_j(1 - t_i) \qquad (4.1)$$

where

s_{ij} = proportion of total equity of the jth firm held by the ith investor ($0 \leq s_{ij} \leq 1$)

d_j = cash dividend paid out by the jth firm.

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15. The bold typeface indicates a random variable. The expected value of the random variable is written as $E(\mathbf{X}_j)$. $\mathbf{X}_j$ is inclusive of any cash inflow of loan funds.

For an Australian tax-paying investor, the after investor-level tax cash flow resulting from the receipt of a franked dividend is given by:[16] cash dividend amount – tax liability on gross dividend + imputation credits utilised, which can be expressed mathematically as:

$$s_{ij}(d_j - D_j t_i + \alpha_i D_j t_{jf}) \tag{4.2}$$

where

D_j = $d_j/(1 - t_{jf})$, the gross dividend paid by the jth firm. It is also equal to the cash dividend (d_j) plus imputation credits distributed. The amount of imputation credits distributed can be determined algebraically as $D_j - d_j = D_j t_{jf}$.

t_{jf} indicates the level of franking of a dividend. $t_{jf} = 0$ under a "classical" tax system or if the dividends are unfranked and t_{jf} = current corporate tax rate (maximum) if the dividends are fully franked. Assuming an opening franking account balance of zero, t_{jf} will equal zero if $(\mathbf{X}_j - P_j - r_f L_j)T \le 0$ and the dividend will be fully franked if $d_j \le (1 - T)(\mathbf{X}_j - P_j - r_f L_j)$. The value of t_{jf} could, of course, be less than the current corporate tax rate, corresponding to the dividend being partially franked.

α_i = for the ith investor, the ratio of imputation credits utilised to the imputation credits received ($D_j t_{jf}$).

For domestic investors, the value of α_i is determined as follows:

$\alpha_i = 1$, if $t_i > 0$ and the investor can utilise all the imputation credits distributed with the dividend. This is necessarily the case if $t_i > t_{jf}$.

$0 < \alpha_i < 1$, if the investor is unable to utilise all the imputation credits distributed with the dividend. This necessarily implies that $0 < t_i < t_{jf}$.

$\alpha_i = 0$, if $t_i = 0$.

Offshore investors can be modelled using equation (4.2) by letting $\alpha_i = t_i$, and under this assumption, equation (4.2) reduces to equation (4.1). Accordingly, equation (4.2) can, by the appropriate choice of α_i, be used to model, for both offshore and domestic investors, an after investor-level tax cash flow resulting from the receipt of a dividend. Equation (4.2) can be more easily handled in the subsequent analysis if it is re-arranged to give:

$$s_{ij}(d_j + \theta_i^d D_j t_{jf})(1 - t_i) \tag{4.3}$$

where

θ_i^d = $(\alpha_i - t_i)/(1 - t_i)$, and can be interpreted as the utilisation factor, applicable to the ith investor, of imputation credits distributed with dividends.

Equation (4.3) models, with the appropriate choice of θ_i^d, the after investor-level tax cash flow resulting from dividend distributions for both the offshore investor case (given by equation (4.1)) and the Australian tax-paying investor case (given by equation (4.2)). For example, if a fully-franked dividend of \$2.03 is paid, then at the current corporate tax rate of 33 percent, this corresponds to $t_{jf} = 0.33$ and a gross dividend of \$2.03/(1 – 0.33) = \$3.03.

16. This is more fully explained in G. Peirson, R. Bird, R. Brown & P. Howard, *ibid.*, p. 457.

The amount of imputation credits distributed to the investor equals \$3.03(0.33) = \$1.00. (The amount of imputation credits distributed is also given by \$3.03 – \$2.03 = \$1.00.) If the investor can fully utilise the \$1.00 of imputation credits then θ_i^d will equal 1. For an offshore investor who cannot utilise imputation credits, θ_i^d will equal 0. However, if an Australian tax-paying investor with a tax rate of 15% can only use the imputation credits to reduce their taxable income by \$0.60, α_i will equal 0.6 and θ_i^d will equal 0.53. Note that the final CAPM does not contain the term θ_i^d and hence the exact form of this term is of little import.

Capital gains are assumed to result from both retained earnings and retained imputation credits. As explained previously, it is assumed that \$1 of retained earnings produces \$1 of capital gain, while \$1 of retained imputation credits results in a capital gain of θ_m^r. In algebraic terms, the increase in the value of the firm due to retained earnings is $\mathbf{X_j}(1-T) + P_jT - L_j(1 + r_f(1-T)) - d_j$. The increase due to retained imputation credits is equal to the proportion θ_m^r of the imputation credits earned less the imputation credits distributed, i.e. $\theta_m^r[(\mathbf{X_j} - P_j - r_fL_j)T - D_jt_{jf}]$. The increase in the value of the firm is then subject to capital gains tax, which obviously requires that only the gain be taxed. This requires that the initial value of equity (S_j) be subtracted from the firm value before the amount of capital gains tax is calculated. The capital gain could, of course, be negative, indicating a capital loss.

To summarise, for an equity holder in the jth firm, the value of the end-of-period wealth therefore consists of:

dividends: $s_{ij}(d_j + \theta_i^d D_jt_{jf})(1 - t_i)$;
capital gains due to retained cash: $s_{ij}[\mathbf{X_j}(1-T) + P_jT - L_j(1 + r_f(1-T)) - d_j - S_j](1 - t_i)$;
capital gains due to retained imputation credits: $s_{ij}\theta_m^r[(\mathbf{X_j} - P_j - r_fL_j)T - D_jt_{jf}](1 - t_i)$; and
the capital component of the cash flow: $s_{ij}S_j$;[17]

which combined yields:

$$s_{ij}[\{\mathbf{X_j}(1-T) + P_jT - L_j(1 + r_f(1-T)) + \theta_i^d D_jt_{jf} + \theta_m^r \mathbf{RIC_j} - S_j\}(1 - t_i) + S_j] \quad (4.4)$$

where

$\mathbf{RIC_j}$ = $(\mathbf{X_j} - P_j - r_fL_j)T - D_jt_{jf}$, the amount of imputation credits retained by the jth firm. Equal to the Australian corporate tax paid less the imputation credits distributed via dividends

S_j = value of equity of the jth firm "now", or time zero, given an uncertain end-of-period cash flow, $\mathbf{X_j}$.

The investor maximises utility by holding the optimal amount of the three different classes of securities. Mathematically, the investor's problem can be represented as follows:

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17. The capital component of the cash flow is required because of the modelling of capital gains tax. This tax is calculated with respect to the current value of equity, S_j, to ensure that only capital **gains** are taxed. To calculate the end-of-period cash flow, however, the capital **gain** must be added to the capital component of the cash flow.

$$\text{maximise } \phi_i(M_i, V_i) \tag{4.5}$$

where
ϕ_i = utility function of the ith investor
M_i = expected value of the ith investor's portfolio
V_i = variance of the value of the ith investor's portfolio.

The ith investor's expected portfolio value, after all taxes, is the sum of the expected values of the amounts invested in the three different classes of security. Thus,[18]

$$M_i = (b_{io} + \sum_j b_{ij}L_j)[1 + r_f(1 - t_i)] + \sum_j s_{ij}[(E(\mathbf{X_j})(1 - T) + P_jT - L_j(1 + r_f(1 - T)) + \theta_i^d D_j t_{jf} + \theta_m^r \mathrm{RIC}_\mathbf{j})(1 - t_i) + S_j t_i] \tag{4.6}$$

where
b_{io} = amount of risk-free government debt held
b_{ij} = proportion of risk-free corporate debt of the jth firm held by the ith investor ($0 \leq b_{ij} \leq 1$).

It can be shown[19]

$$V_i = \sum_j \sum_k s_{ij}s_{ik}(1 - T')^2(1 - t_i)^2 \mathrm{cov}(\mathbf{X_j}, \mathbf{X_k}) \tag{4.7}$$

where
$T' = (1 - \theta_m^r)T$, and can be interpreted as an "effective" tax rate.

Optimisation of equation (4.5), given equations (4.6), (4.7) and an additional budget constraint, results in the following equation which must be satisfied to ensure an optimum is achieved[20]:

$$E(\mathbf{X_j})(1 - T) + P_jT - L_j(1 + r_f(1 - T)) + \theta_i^d D_j t_{jf} + \theta_m^r \mathrm{RIC}_\mathbf{j} - S_j(1 + r_f) = \gamma_i(1 - t_i)(1 - T')^2 \sum_k s_{ik} \mathrm{cov}(\mathbf{X_j}, \mathbf{X_k}) \tag{4.8}$$

where
γ_i = risk aversion of the ith investor
= $-(2\partial\phi_i/\partial V_i)/(\partial\phi_i/\partial M_i)$.

The left-hand side of equation (4.8) represents the expected value, measured on a before investor-level tax basis, of an investment in the jth firm less the cash flow that could be

18. More formally, the notation should be $E(\mathbf{RIC_j})$, rather than $\mathbf{RIC_j}$, as $\mathbf{RIC_j}$ is a random variable.
19. This equation is valid as equation (4.4) can be rewritten to ensure that the only stochastic variable in the equation is the uncertain end-of-period cash flow $\mathbf{X}_j$. The subscript k denotes a representative (kth) firm.
20. The optimisation follows the approach taken by Ashton. Proof of equation (4.8) is available from the author on request.

anticipated from investing the same amount (S_j) in risk-free assets. It represents the premium that is expected to accrue to the ith investor for bearing risk. The right-hand side of the equation quantifies the level of risk borne by the ith investor. That is, equation (4.8) specifies the additional return required by the ith investor for bearing risk.

5. Market Equilibrium

If it is assumed that all investors have optimised their portfolios (i.e. all investors' portfolios are in equilibrium) and that the market is in equilibrium (i.e. all available securities are priced such that they are held by investors), then expressions can be found for the value of equity in the jth firm, and for the aggregate levels of debt and equity in the market.

Mathematically, this can be represented by summing equation (4.8) over all the investors in the jth firm, and using the market clearing condition that all the equity of the jth firm is held by someone (i.e. $\sum_i s_{ij} = 1$). Because it has been assumed that investors have homogenous expectations and information is simultaneously available to all investors at zero cost, $E(\mathbf{X_j})$ is the same for all investors and hence can be taken outside the summation.[21]

Summing equation (4.8) over all the investors in the jth firm results in:

$$E(\mathbf{X_j})(1 - T) + P_jT - (1 + r_f(1 - T))L_j + \theta_j^d D_j t_{jf} + \theta_m^r RIC_j - S_j(1 + r_f)$$
$$= \chi(1 - T')^2 \sum_k \text{cov}(\mathbf{X_j},\mathbf{X_k}) \tag{5.1}$$

where

$$\frac{1}{\chi} = \sum_i \left(\frac{1}{\gamma_i(1 - t_i)}\right)$$

$$\theta_j^d = \sum_i \left(\frac{\theta_i^d}{\gamma_i(1 - t_i)}\right) \Big/ \frac{1}{\chi}.$$

Equation (5.1) is an expression for the value of equity of the jth firm (S_j) given an uncertain end-of-period cash flow ($\mathbf{X_j}$), assuming that all investors are in equilibrium and that the market is in equilibrium.[22]

Consistent with Ashton's interpretation, χ can be considered as the harmonic mean of the individual's risk aversion, weighted by one minus the investor-level tax rate. θ_j^d can be

21. Although $E(\mathbf{X_j})$, the expected before corporate tax cash flow, is assumed to be the same for all investors, the fact that the tax rate and tax deductions are assumed to be known implies that investors agree on returns after corporate tax, before investor-level taxes. This is consistent with the approach taken by Brennan and Ashton.

22. It is noted that in summing across all investors in the jth firm a weighted average utilisation factor for imputation credits is derived. In general, this utilisation factor will not correspond to any single individual's ability to utilise imputation credits. It has been noted by an anonymous referee that this may imply the resulting equilibrium prices may not be appropriate for any single individual for any given asset.

interpreted as the weighted market-average value of θ_i^d (the utilisation factor of distributed imputation credits), with weights inversely proportional to the individual's risk aversion and one minus the investor-level tax rate. The form of the equations specifying χ and θ_j^d means that risk-averse, high tax rate individuals (i.e. the "marginal investor") will tend to strongly influence the values of χ and θ_j^d.

An expression can also be obtained for the aggregate market values of debt and equity by summing equation (5.1) over all (j) firms. That is,

$$\sum_j [E(\mathbf{X_j})(1-T) + P_jT - (1 + r_f(1-T))L_j + \theta_j^d D_j t_{jf} + \theta_m^r RIC_j] - S_m(1 + r_f)$$
$$= \chi(1-T')^2 \sum_j \sum_k \operatorname{cov}(\mathbf{X_j}, \mathbf{X_k}) \tag{5.2}$$

where
$S_m = \sum_j S_j$, the total value of equity of all firms.

6. Cost of Equity Capital and Expected Returns

The cash flow $E(\mathbf{X_j})(1-T) + P_jT - L_j(1 + r_f(1-T))$ is after corporate tax and debt repayments but before investor-level tax and is, therefore, the before investor-level tax cash flow to equity holders. The expected return on equity is thus the expected return from this cash flow; equivalently it can be thought of as the cost of equity capital:

$$E(\mathbf{r_j}) = \frac{E(\mathbf{X_j})(1-T) + P_jT - (1 + r_f(1-T))L_j - S_j}{S_j} \tag{6.1}$$

where
$\mathbf{r_j}$ = return on the jth firm, before investor-level taxes, due to **cash flow** only ($\mathbf{r_j}$ is a random variable).

A similar expression, defined in terms of cash flow only, for the expected return on the market is:

$$E(\mathbf{r_m}) = \frac{\sum_j [E(\mathbf{X_j})(1-T) + P_jT - (1 + r_f(1-T))L_j] - S_m}{S_m}. \tag{6.2}$$

The expected returns on the jth firm can be related to equation (5.1), which is an expression for the jth firm assuming all investors are in equilibrium and that the market is in equilibrium. Combining equations (5.1) and (6.1) results in:

$$E(\mathbf{r_j}) - r_f + \theta_j^d D_j' t_{jf} + \theta_m^r RIC_j' = \frac{\chi(1-T')^2 \sum_k \operatorname{cov}(\mathbf{X_j}, \mathbf{X_k})}{S_j} \tag{6.3}$$

where

D_j' $= D_j/S_j$, the gross dividend yield of the jth firm
RIC_j' $= RIC_j/S_j$, the "yield" associated with amount of the retained imputation credits.

By combining equations (5.2) and (6.2), an expression similar to equation (6.3) can be obtained for the market return, and eliminating $\chi(1 - T')^2$ from that equation and equation (6.3) results in:

$$E(\mathbf{r_j}) + \theta_j^d D_j' t_{jf} + \theta_m^r RIC_j' = r_f + \beta_j (E(\mathbf{r_m}) + \theta_m^d D_m' t_{mf} + \theta_m^r RIC_m' - r_f) \quad (6.4)$$

where

$\theta_m^d D_m' t_{mf}$ $= \sum \theta_j^d D_j t_{jf} / S_m$ and is the sum of; imputation credits distributed by the jth firm, multiplied by the proportional value of distributed imputation credits applicable to the jth firm, all divided by the equity value of the market and can be interpreted as an effective market average yield of distributed imputation credits

RIC_m' $= \sum RIC_j/S_m$ and is the sum of; imputation credits retained by the jth firm divided by the equity value of the market and can be interpreted as a market average yield of retained imputation credits

$$\beta_j = \left(\frac{S_m}{S_j}\right) \frac{\sum\limits_k \mathrm{cov}(\mathbf{X_j}, \mathbf{X_k})}{\sum\limits_j \sum\limits_k \mathrm{cov}(\mathbf{X_j}, \mathbf{X_k})} .$$

The last equation can be shown to be equivalent to:

$$\beta_j = \frac{\mathrm{cov}(\mathbf{r_j}, \mathbf{r_m})}{\mathrm{var}(\mathbf{r_m})}.$$

That is, a firm's beta is unchanged by the introduction of the dividend imputation tax system, assuming that returns (given by equations (6.1) and (6.2)) continue to be defined and measured in terms of cash flow only.

Equation (6.4) is the cost of equity capital for the jth firm. It is recognisable as the "classical" CAPM with four additional terms:

$$\theta_j^d D_j' t_{jf},\ \theta_m^r RIC_j',\ \theta_m^d D_m' t_{mf} \text{ and } \theta_m^r RIC_m'$$

which can be interpreted as follows. The term $\theta_j^d D_j' t_{jf}$ is specific to the jth firm and represents the value of the imputation credits distributed by the firm, divided by the value of the firm's equity. In effect, it is the rate of return (or yield) attributable to distributed imputation credits. Similarly, the term $\theta_m^r RIC_j'$ is also firm-specific and represents the rate of return attributable to retained imputation credits. The remaining terms $\theta_m^d D_m' t_{mf}$ and $\theta_m^r RIC_m'$ are "market level" terms which mirror the firm-specific terms, the first referring to distributed imputation credits and the second to retained imputation credits. All four terms are, of course, tax-adjusted.

Equation (6.4) can be interpreted as the "classical" CAPM where (total) returns are defined in terms of cash flow ($\mathbf{r_j}$ and $\mathbf{r_m}$), distributed imputation credits ($\theta_j^d D_j' t_{jf}$ and $\theta_m^d D_m' t_{mf}$) and retained imputation credits ($\theta^r_m RIC'_j$ and $\theta^r_m RIC'_m$). That is, where the total return on the jth firm is given by: $E(\mathbf{r_j}) + \theta^d_j D'_j t_{jf} + \theta^r_m RIC'_j$, and the total return on the market is given by: $E(\mathbf{r_m}) + \theta^d_m D'_m t_{mf} + \theta^r_m RIC'_m$, equation (6.4) is simply the "classical" CAPM.

7. The Effect of Dividend Imputation on the Risk Premium

The application of the CAPM requires an estimate of the future risk premium, $E(\mathbf{r_m}) - r_f$, and one forecasting technique is to use a historically measured risk premium. However, it is likely that the data used will relate to a period before the introduction of the imputation system. An anonymous referee has suggested that the risk premium could have altered with the introduction of dividend imputation, as the tax rate on one component, $E(\mathbf{r_m})$, has altered vis-a-vis the tax rate on r_f. Although a difficult issue to resolve, it is suggested that the introduction of dividend imputation should not have caused a significant change in the estimated value of the "new" risk premium, $E(\mathbf{R_m}) - r_f$, where $\mathbf{R_m}$ is now given by:

$$\mathbf{R_m} = \mathbf{r_m} + \theta^d_m D'_m t_{mf} + \theta^r_m RIC'_m \tag{7.1}$$

and can be interpreted as the total market return, including the returns resulting from the utilisation of imputation credits.[23, 24]

Combining equations (6.4) and (7.1) therefore results in the final CAPM which can be written as:

$$E(\mathbf{r_j}) = r_f + \beta_j [E(\mathbf{R_m}) - r_f] - \theta^d_j D'_j t_{jf} - \theta^r_m RIC'_j. \tag{7.2}$$

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23. The main elements of the argument are as follows. Historically, and in this paper, $\mathbf{r_m}$ is defined in terms of cash flow only. The introduction of dividend imputation has, however, ensured that market participants have obtained additional returns via the flow of imputation credits, either by dividend distributions or by (additional) capital gain because of the expectation of the distribution of imputation credits at some later date. Thus if the total market return ($\mathbf{R_m}$) is defined to include the utilisation of imputation credits, the ("grossed-up") return on the market now consists of three elements: the return due to cash flow ($\mathbf{r_m}$), the return due to distributed imputation credits $\theta^d_m D'_m t_{mf}$ and the return due to retained imputation credits ($\theta^r_m RIC'_m$).
There is no reason to expect the total risk premium, now equal to $E(\mathbf{R_m}) - r_f$, to have changed with the introduction of dividend imputation, since it results from the risk aversion of investors, upon which the introduction of dividend imputation has had no obvious effect. That is, investors will still require the same returns, net of all taxes, for bearing risk both before and after the introduction of dividend imputation. The assumption that the total risk premium has remained unchanged implies that the individual terms that make-up $\mathbf{R_m}$ do not have to be evaluated. Rather, the total return on the market has to be re-interpreted to include the net, or cash, return (consisting of income and capital gains) plus the return obtained, or expected to be obtained, from imputation credits.
24. R.R. Officer in "The Australian Imputation System for Company Tax and its Likely Effect on Shareholders, Financing and Investment", unpublished, dated 7 November 1989, suggests that there will not be a significant change in the measured risk premium, although he notes that the risk premium could be marginally reduced over time as companies replace debt with equity in their capital structure due to equity being a more attractive form of financing under the dividend imputation tax system than under the "classical" tax system. The crucial question of course, is whether the measured risk premium includes the value associated with imputation credits.

This discount rate should be applied to cash flows after corporate tax, before investor-level taxes.

As an aside, the Australian Stock Exchange ("ASX") All Ordinaries Accumulation Index, which is the usual method of measuring $\mathbf{r_m}$ in Australia, is defined in terms of cash, or net, dividend yield and capital gains, and ignores the value associated with imputation credits distributed with dividends. Capital gains will, however, occur from retained cash and retained imputation credits. Hence, while the market return measured by the ASX All Ordinaries Accumulation Index will capture the effect of retained imputation credits, it ignores the effect of distributed imputation credits.

8. Discussion of the CAPM

This section discusses the effects of the dividend imputation tax system on a firm's cost of equity capital. Also examined is an alternative definition of "cash flow" and discount rate which is shown to be equivalent to the definition of cash flow and discount rate as given by equation (7.2).

Before commenting in detail, it should be noted that the CAPM, as given by equation (7.2), involves two terms, θ_j^d and θ_m^r, which cannot readily be measured. In a practical application, an analyst could estimate the values of these terms on the basis of "market experience" and an assessment of the investor base of the firm. While the values of θ_j^d and θ_m^r must ultimately be determined empirically, in most of the comments that follow it has been assumed that both θ_j^d and θ_m^r are greater than zero.

The CAPM, as given by equation (7.2), has the following characteristics:

- The CAPM has a form similar to the CAPM developed by Brennan. Whereas the Brennan CAPM predicts a higher cost of equity capital for a firm paying out dividends, due to the comparative tax disadvantage of dividends to end-investors, the CAPM derived in this paper predicts a lower cost of equity capital for a firm generating imputation credits, due to the tax advantages of imputation credits to end-investors.

- If the firm is to invest in an offshore project no imputation credits will be earned, as no Australian corporate tax will be paid, and hence the appropriate discount rate to use in this situation is the "classical" discount rate. Using the notation developed in this paper, $t_{jf} = RIC_j = 0$ and hence equation (7.2) reverts to the "classical" CAPM.

- The cost of equity capital for an Australian tax-paying firm, applicable to an after corporate tax, before investor-level tax cash flow, is lower than if the same cash flow were generated by a firm which did not pay Australian tax, due, say, to the presence of carried forward tax losses. In the latter case, $t_{jf} = RIC_j = 0$ and the CAPM reverts to the "classical" CAPM.[25]

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25. A worked example that is available from the author on request shows that the cost of equity capital for an Australian tax-paying firm is 11.8% versus 16% for the non tax-paying firm.

- The cost of equity capital for an Australian tax-paying firm with predominantly Australian investors is lower than if the same firm were owned by offshore investors. In the latter case, $\theta^d_j = \theta^r_m = 0$ and the CAPM reverts to the "classical" CAPM.[26]

- The CAPM of equation (7.2) predicts that, ceteris paribus, a firm which pays a "higher" yield of imputation credits ($D'_j t_{jf} = D_j t_{jf}/S_j$) will have a lower cost of capital. Whilst mathematically a firm which has a high "yield" of retained imputation credits ($RIC'_j = RIC_j/S_j$) will, ceteris paribus, also have a lower cost of capital, the amount of retained imputation credits is obviously a function of the dividend policy of the firm.

The optimal dividend policy of the firm, within the framework of this paper, is for the firm to distribute all of its imputation credits because it is expected that θ^r_m will be less than θ^d_j. That is, imputation credits distributed "next year" will be worth less, in present value terms, than imputation credits distributed today, assuming that shareholders in the jth firm can utilise distributed imputation credits to the same extent as the market-average utilisation factor for retained imputation credits, both now and in the future. For a firm that does not distribute all its imputation credits, the increase in the cost of equity capital is equal to $(\theta^d_j - \theta^r_m)RIC_j/S_j$.[27]

The CAPM as given by equation (7.2) is applicable to a cash flow after corporate tax and debt repayments but before investor-level taxes. An equivalent result, which is perhaps more easily interpreted, can be obtained by re-defining both the "cash flow" that is to be discounted and the discount rate. This can be seen by considering a firm with an expected end-of-period cash flow, measured after corporate tax and debt repayments but before investor-level taxes, given by $E(\mathbf{X}_j)(1-T) + P_jT - (1 + r_f(1-T))L_j$. Applying equation (7.2) to this cash flow to solve for the current value of equity (S_j) results in, after some re-arranging:

$$S_j = \frac{E(\mathbf{X}_j)(1-T) + P_jT - (1 + r_f(1-T))L_j + \theta^d_j D_j t_{jf} + \theta^r_m RIC_j}{1 + r_f + \beta_j(E(\mathbf{R_m}) - r_f)}. \qquad (8.1)$$

The denominator of equation (8.1) is recognisable as one plus the "classical" cost of equity capital, and given the argument developed in Section 7, can be calculated using the historical risk premium. The numerator is recognisable as the after corporate tax cash flow with two additional terms, discussed in Section 6, reflecting the additional value accruing to investors due to the distribution, or expected distribution, of imputation credits. That is, by re-defining the "cash flow" to include the cash, or market, value of both distributed and retained imputation credits, the revised definition of "cash flow" can be discounted at the "classical" cost of equity capital.

It is also useful to consider a special case of equation (8.1). Assume that the firm distributes all of its imputation credits ($RIC_j = 0$) and assume also that all investors can fully utilise the

26. A worked example that is available from the author on request shows that the cost of equity capital for an "Australian" firm is 11.8% versus 16% for the "foreign" firm.

27. Offsetting this effect is the fact that the tax rate on capital gains is less than the tax rate on income and hence this estimate of "value loss" can be considered a worst case estimate.

imputation credits received from dividends ($\theta_j^d = 1$). Using the fact that $D_j t_{jf} + RIC_j$ = the Australian corporate tax paid by the jth firm, equation (8.1) becomes:

$$S_j = \frac{E(\mathbf{X}_j) - (1 + r_f) L_j}{1 + r_f + \beta_j (E(\mathbf{R}_m) - r_f)}. \tag{8.2}$$

Alternatively, equation (8.2) will result if it is assumed that all investors can fully utilise imputation credits, whether distributed or retained ($\theta_j^d = \theta_m^r = 1$). Equation (8.2) shows that, under these restrictive assumptions, the value of an uncertain future cash flow is independent of the Australian corporate tax rate and of the amount of allowable tax deductions. Further, it implies that the value to shareholders of a future cash flow to be received by their firm can be found by discounting the before Australian corporate tax cash flow using one plus the "classical" discount rate. Hence, under these assumptions, the imputation tax system has essentially eliminated the effect of Australian corporate tax on the valuation of cash flows.

At the other extreme, Officer[28] has argued that as the Australian economy is relatively open with respect to capital flows, the introduction of dividend imputation is unlikely to cause a significant change in a firm's after corporate tax cost of capital. Officer's argument is based on the assumption that as real, risk-adjusted rates of return will be approximately equivalent among those countries with open economies and no change has occurred elsewhere, the after corporate tax return required in Australia will be unchanged. Note that application of this to expected real returns requires that strong assumptions be made about the applicability of the international parity conditions. Intuitively, if the marginal (price-setting) investors are offshore investors to whom imputation credits are valueless, a dividend imputation tax system will have no effect. Officer has essentially assumed, within the framework of this paper, that $\theta_j^d = \theta_m^r = 0$ and hence the value to shareholders of a future cash flow to be received by their firm can be found by discounting the after Australian corporate tax cash flow at the "classical" discount rate. Most likely, the truth is somewhere between the two approaches set out above.

9. Conclusion

This paper set out to derive a CAPM consistent with a cash flow measured after corporate tax but before investor-level taxes in the context of the Australian dividend imputation tax system. This question has considerable significance since it relates directly to the issue of specifying the discount rate that firms in Australia should use to value risky cash flows. The answer derived in this paper is given by equation (7.2) which presents a CAPM that contains two terms that capture the effects of the dividend imputation tax system. Unfortunately these terms contain two parameters, θ_j^d and θ_m^r, which cannot readily be measured although their theoretical meaning is apparent. The agenda for future research includes estimating empirically the values of these parameters and determining if the

28. R.R. Officer, "A Note on the Cost of Capital and Investment Evaluation for Companies under the Imputation Tax", *Accounting and Finance*, November 1988, pp. 65–71.

utilisation factor for retained imputation credits (θ^r_m) is in fact constant for all firms in the market as has been assumed in this paper.

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