Estimation of, and correction for, biases inherent in the Sharpe CAPM formula

A report for the Energy Networks Association Grid Australia and APIA

Dr. Tom Hird
Professor Bruce Grundy
Daniel Young

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Executive Summary

Terms of reference

1. CEG has been asked to develop an empirical study of the sensitivity of the return on Australian equity to the beta of the equity. We have been asked to specifically test whether the actual sensitivity to beta is equal to the sensitivity implied by the Sharpe (1964) CAPM model using the government bond rate as the risk free rate – as set out in 6A.6.2 (b) of the National Electricity Rules (NER).

2. We have also been asked to attempt to explain whether our results are consistent with both:

   - empirical results of similar studies in other equity markets around the world;
   - modern asset pricing theory, including developments in the CAPM since Sharpe first outlined his model in 1964.

Key findings

It is an empirical regularity found in the US and elsewhere that the actual sensitivity of equity returns to beta is lower than implied by the Sharpe CAPM using the Government bond rate as the risk free rate.

Analysis of monthly Australian market data from 1964 to 2007 confirms that the same is true in Australia.

This finding is consistent with refinements to the CAPM since Sharpe's 1964 paper.

These results create a strong presumption for the adoption of an equity beta of close to 1.0 when using the Sharpe CAPM formula – even if observed values for equity beta are materially different to 1.0.

A failure to adopt this recommendation will, other things equal, deny regulated businesses a reasonable opportunity to recover their efficient costs.

State of play

3. Currently, the National Electricity Rules (NER) prescribes the use of the following formulation of the Sharpe CAPM formula[1] to calculate the required equity return (k_e):

$$Sharpe\ CAPM, \qquad k_e = R_f + \beta_e \cdot (R_m - R_f) \qquad \text{(i)}$$

where: R_f is the risk free rate (proxied in the NER in real terms by the prevailing yield on nominal Commonwealth Government bonds less an estimate of expected inflation);

β_e is the equity beta (currently prescribed at a value of 1.0 for electricity transmission in the NER[2]); and

$R_m - R_f$ is the expected market risk premium (MRP) being the expected return on the market less the risk free rate (the MRP is prescribed at a value of 6% for electricity transmission in the NER[3])

4. In this paper we refer to this formulation, with the risk free rate proxied by the government bond rate, as the Sharpe CAPM formula. However, we note that the risk free asset in the Sharpe CAPM is not necessarily the government bond rate but is rather a zero beta asset. The Australian Energy Regulator is running a consultation process on, amongst other things, whether there is a need to amend the parameter values used in the above equation.

The question

5. The approach set out in the NER broadly reflects precedent set by jurisdictional regulators (including in terms of the use of the above formula and the prescribed parameter values). However, some jurisdictional regulators have argued that the best estimate of observed equity betas for comparable firms is less than 1.0. Most notably, the Essential Services Commission of Victoria (ESCV) has argued

[1] The AER does not refer to the Sharpe CAPM but only to the CAPM. However, the AER does define the CAPM at page 6 of the discussion paper using the formula derived by Sharpe in 1964.

[2] See section 6A.6.2 Return on Capital of the National Electricity Rules (NER).

[3] See section 6A.6.2 Return on Capital of the National Electricity Rules (NER).

that the best estimate of the equity beta for firms comparable to Victorian gas distributors is 0.7.[4]

6. Even if one were to accept that 'the best' estimate of the beta was less than 1.0 it does not follow that the best estimate of the cost of equity is estimated using a beta value of 0.7 in the Sharpe CAPM formula.

7. In our view it is a major failing of the current AER discussion paper that it spends 11 pages discussing the equity beta and spends zero pages discussing whether the formula in which equity beta is used is accurate.

8. The purpose of this report is to empirically estimate the actual relationship between beta and the cost of equity in the Australian market and to compare this with the Sharpe CAPM formula.

The answer – the Sharpe CAPM gives biased estimates of the cost of equity

9. Our conclusion is that the Sharpe CAPM does not adequately describe how capital markets actually set firms' required equity returns. Theoretical and empirical advances since 1964 have demonstrated that the Sharpe CAPM formula results in biased estimates of the required returns actually determined in capital markets. Specifically, the Sharpe CAPM underestimates the required return on equity with $\beta_e < 1$ and overestimates the required return on equity with $\beta_e > 1$.

Theoretical refinement

10. The Sharpe CAPM formula is based on a number of unrealistic assumptions about investors and capital markets. In particular, the derivation of this formula relies on the assumption that:

    i. Investors can borrow at the Government bond rate to invest in risky equities; and

    ii. Investors invest once, hold that portfolio unchanged for a given period, and then consume their entire wealth at the end of that period. In the terminology of finance theory the Sharpe CAPM is a 'single period' model.

11. Sharpe (1964) himself states in relation to his assumptions:

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[4] ESCV GAAR decisions (Draft and Final). See http://www.esc.vic.gov.au/public/Energy/Regulation+and+Compliance/Decisions+and+Determinations/Gas+access+arrangements+review+2008-12/Gas+access+arrangement+review+2008+12.htm

> *"Needless to say, these are highly restrictive and undoubtedly unrealistic assumptions."*

12. Since 1964 the CAPM has undergone considerable refinement and both of these unrealistic assumptions have been relaxed. The Black CAPM (1972), developed by the father of the Black-Scholes option pricing model Fischer Black, retains all the other assumptions of the Sharpe CAPM but assumes that equity investors can only borrow at a rate above the Government bond rate (ie, relaxes only assumption i) above). With this more realistic assumption in place, the Black CAPM derives a formula the cost of equity is given by: $\beta_e < 1$

$$Black\ CAPM, \quad k_e = R_f + \propto + \beta_e \cdot (R_m - R_f - \propto) \qquad \text{(ii)}$$

where: a is a positive constant

13. That is, when the Sharpe CAPM is adjusted to take into account the real world fact that investors cannot borrow at the Government bond rate to negatively gear investments in risky equities then, as a matter of correct logic and finance theory, two things follow:

    a. equities that have a zero beta will earn more than the government bond rate (by "α" in the above equation); and

    b. the sensitivity of required returns to beta will be lower $(R_m - R_f - \propto)$ instead of $(R_m - R_f)$ as predicted in the Sharpe CAPM.

14. Similarly, the unique role of equity beta in the Sharpe CAPM flows directly from the extreme simplifying assumption that investors only invest for a single period (assumption ii) above). Of course, the reality is that investors invest over their entire life and are interested not just in the returns on their portfolio in the next period but returns on their portfolio over their entire life. The Merton CAPM (1973), developed by Nobel Prize winner Robert Merton, showed that in multiple period models of the CAPM factors other than β_e drive equity returns. In particular, investors also care about the correlation between returns in this period and the profitability of reinvesting those returns in the next period (reinvestment opportunities). Equity that pays off more when reinvestment opportunities are so attractive that investors would otherwise cut-back consumption in order to invest more will be more valuable (lower risk) than equity that pays off less when reinvestment opportunities are high. The 'technology bubble' and the 'commodity boom' could reasonably fit into these categories.

15. Unlike beta, covariance with reinvestment opportunities is very difficult to measure directly (as it is difficult to measure *ex-ante* perceived reinvestment

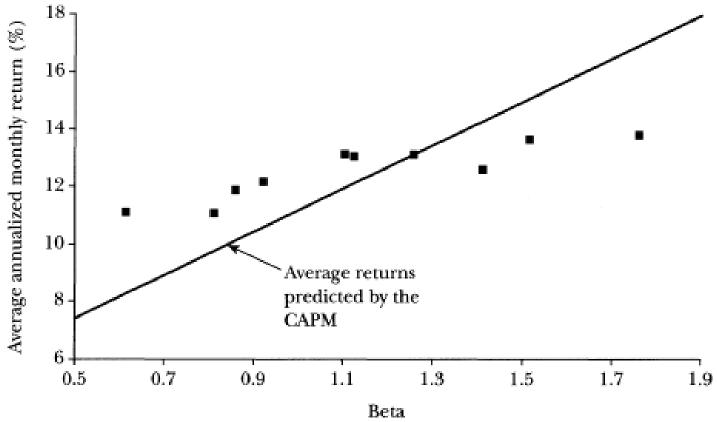
opportunities). However, researchers have used the Merton CAPM (also known as the intertemporal CAPM) as the rationale for testing for proxies for this risk. Most famously, Fama and French (2004) have described their three factor model as a practical implementation of the Merton CAPM. The Fama and French three factor model is the model that best predicts the returns that are actually observed in capital markets.

Empirical refinement

16. There have been a number of empirical tests of the Sharpe CAPM in different countries at different times and a near universal finding of these tests is that the Black CAPM outperforms the Sharpe CAPM. That is, the Sharpe CAPM overestimates the sensitivity of equity returns to beta and will underestimate the required returns set in capital markets on stocks with equity betas of less than 1.0.

17. This is depicted in the figure below from Fama and French (2004). The figure shows clearly the difference between the actual relation between a stock's beta and its return compared to the relation predicted by the Shape CAPM.

Figure 2
Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



18. In the above graph, the Government bond rate defines the intercept of the CAPM security market line (SML = the upward-sloping line). The slope of the line is defined by the market risk premium measured relative to the Government bond rate. That is, the SML as drawn is of the form described by the Sharpe CAPM.

19. As is clear from the above graph the actual relationship between beta and market returns is much flatter than that predicted by the Sharpe CAPM. This is a general finding of the empirical tests of the CAPM as described by Fama and French.

> *"The Sharpe-Lintner CAPM predicts that the portfolios plot along a straight line, with an intercept equal to the risk-free rate, R_f, and a slope equal to the expected excess return on the market, $E(R_M)$- R_f. We use the average one-month Treasury bill rate and the average excess CRSP market return for 1928-2003 to estimate the predicted line in Figure 2. Confirming earlier evidence, the relation between beta and average return for the ten portfolios is much flatter than the Sharpe-Lintner CAPM predicts. The returns on the low beta portfolios are too high, and the returns on the high beta portfolios are too low. For example, the predicted return on the portfolio with the lowest beta is 8.3 percent per year; the actual return is 11.1 percent. The predicted return on the portfolio with the highest beta is 16.8 percent per year; the actual is 13.7 percent."*

20. Precisely the same relationship has been found in every study of this kind that we are aware of. The seminal studies of this kind were performed by Fama and Macbeth (1973) and Black, Jensen and Scholes (1972). In relation to more recent tests Fama and French (2004) state:

> *"Fama and French (1992) also confirm the evidence (Reinganum, 1981; Stambaugh, 1982; Lakonishok and Shapiro, 1986) that the relation between average return and beta for common stocks is even flatter after the sample periods used in the early empirical work on the CAPM."*

21. More recently, Campbell and Vuolteenaho (2004) have estimated that the return on zero beta equity is above not only the government bond rate but also is above the market return. That is, lower equity betas are actually associated with higher returns rather than the opposite as predicted by the single period CAPM models (Sharpe and Black). The guest finance professor at the 2008 ACCC regulatory conference, Professor Jagannathan, also presented results of his own studies that confirmed this relationship.[5]

22. We note that the AER discussion paper references the classic Fama and Macbeth (1973) and Black Jensen Scholes (1972) studies (see footnote 134 on page 58 of the discussion paper). However, it only does so only to support of the claim that:

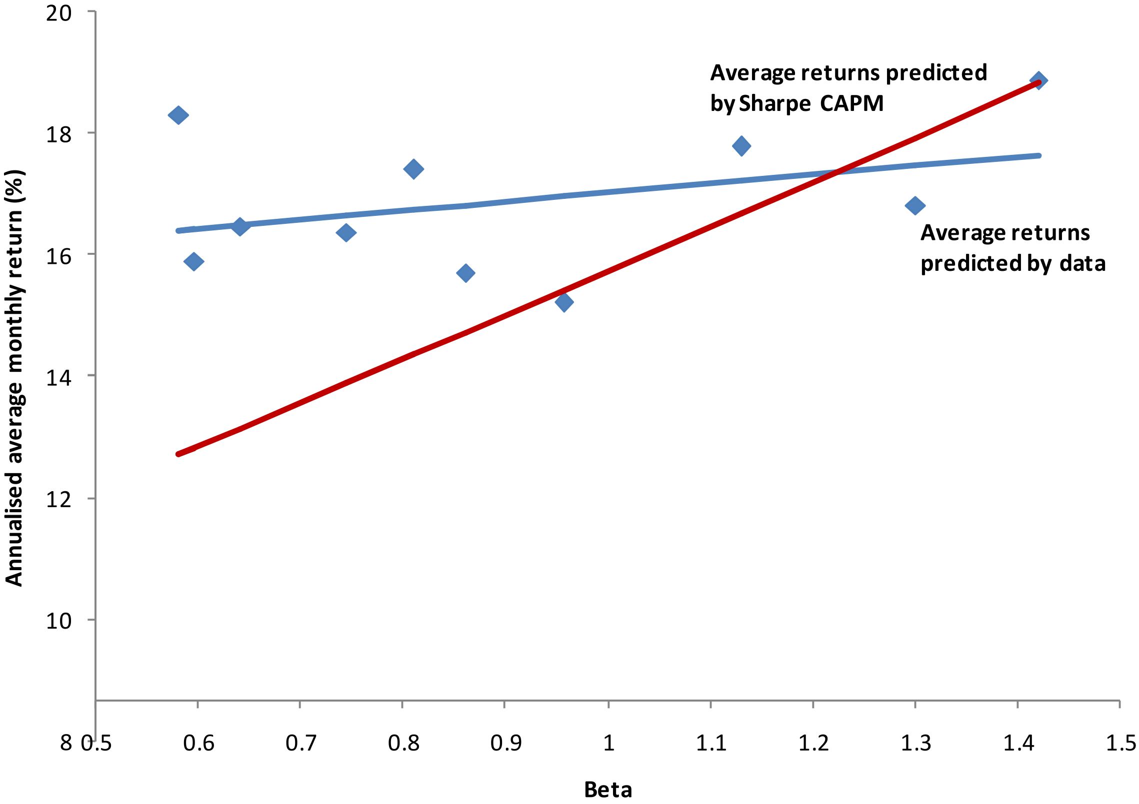
[5] See slide 26 at http://www.accc.gov.au/content/item.phtml?itemId=836242&nodeId=d94f0f6388fac76502937d425a2285f7&fn=Ravi%20-%20CAPM%20Debate%20July11th2008.pdf

"A five-year period is usually regarded as an appropriate trade-off between the number of observations and the stability of the equity beta estimate when monthly data is being used."

23. We are surprised that the AER would reference these two seminal papers only with respect to how they estimated beta. In our view, the AER would be well advised to carefully review the actual conclusions of these papers. As described above, both Fama and Macbeth (1973) and Black Jensen Scholes (1972) find beta does not play the role predicted by the Sharpe CAPM. This is the key finding of these seminal studies and it is the finding of most relevance to the AER in the context of its current WACC review.

24. For the purpose of this report we have replicated the Fama and Macbeth study using 44 years of monthly Australian return data from 1964 to 2007. We also find the same results as other researchers. The figure below summarises the key empirical results of our study.

Figure 1: Sharpe CAPM Predictions vs Actual Relationship in Australian Capital Markets. 300 largest stocks with return data from 1964 to 2007 (equal weighting to each stock)



25. Consistent with the findings of other researchers in other markets, our results find that the Black CAPM formula is superior to the Sharpe CAPM in terms of predicting the required returns on equity. As summarised in the above graphic, restricting our sample to data for the 300 largest equities from 1964 to 2007, zero beta equity tended to earn around 16% pa over that period (8.1% per annum more than the return on government bonds). This is a robust statistical result: the expected return on zero beta equity is statistically significantly greater than the rate on government bonds at the 99.7% confidence level[6].

6 That is, based on the Australian data for the 300 largest firms we can be 99.7% certain that zero beta equity will earn more than the risk free rate. That is, we can be 99.7% confident that the Black CAPM is a better description of reality than the Sharpe CAPM.

26. As described in the body of our report, these results are not sensitive to the use of only the 300 largest stocks in the data set. That is, no matter how one 'cuts the data' the same result is found – zero beta equity earn significantly more than the government bond rate. This result is a direct contravention of the Sharpe CAPM formula when assuming the risk free rate is proxied by the government bond rate.

27. We also find that the estimated sensitivity of market returns to beta (the slope of the average returns predicted by the data) is much lower than predicted by the Sharpe CAPM (and is not statistically significantly different from zero).

The solution

28. Given that both theory and empirics clearly demonstrate that naïve application of the Sharpe CAPM gives biased estimates of the cost of equity, the AER is faced with two options for addressing this bias:

    i. adjust the current NER CAPM formula to allow zero beta equity to earn returns above the government bond rate ;or

    ii. set the value of β_e in the current NER formula closer to one than is implied by market estimates of β_e.

29. Implemented consistently either approach will give the same result. Consequently, which of these options is adopted is a matter of form and not substance (ie, we equally recommend either to the AER).

Adopting a different equation

30. The bias in equation (i) can be corrected by simply adding an appropriate positive amount 'α' to the risk free rate and deducting this amount from the MRP (as would be done under the Black CAPM). That is, the required equity return would be equal to:

$$R_f + \propto + \beta_e \cdot (R_m - R_f - \propto) \qquad \text{(ii)}$$

31. In this formulation, consistent with theory and empirics, equity with zero β_e earns on average more than the risk free rate and, compared to equation (i), required returns increase more slowly as β_e increases. Based on our Australian empirical work the best estimate of "α" is approximately the same as $(R_m - R_f)$. That is, there does not appear to be any significant relation between β_e and equity returns in the Australian market. If we conservatively adopt a value for "α" that is 1% less

than the value of $(R_m - R_f)$ and the current NER value of 6% for $(R_m - R_f)$ is retained, this would imply a value for "α" of 5% and equation (ii) would be:

$$R_f + 5\% + \beta_e \cdot 1\%$$

32. Thus, if the best estimate of the value of β_e was 0.5 then the estimated cost of equity would be 5.5% in excess of the risk free rate (compared to 3.0% in excess of the risk free rate using the Sharpe CAPM formula).

Adopting a different value of β_e

33. Alternatively, if the current NER formula is retained then the equity beta inserted into that equation would need to be determined from the observed equity beta $(\beta_e{}^{\text{observed}})$ in the following manner:

$$\beta_e{}^{\text{used in Sharpe CAPM formula}} = \frac{\propto}{R_m - R_f} + \beta_e^{\text{observed}} \cdot (1 - \frac{\propto}{R_m - R_f})$$

34. β_eThe effect of this is to use a beta closer to 1.0 than would be the case if one simply relied on the observed β_e. This is mathematically equivalent to simply adopting equation (ii). For example, if we adopt the same parameter values (α = 5%, $(R_m - R_f)$ = 6% and $(\beta_e{}^{\text{observed}})$ = 0.5) then we get a value for beta to be used in equation (i) of 0.916 and this results in an estimated cost of equity 5.5% in excess of the risk free rate (as was the case above).

Consistency with NER

35. We note that 6A.6.2 (j) (1) requires the AER to perform the current review having regard to:

> *"the need for the rate of return calculated for the purposes of paragraph (b) to be a forward looking rate of return that is commensurate with prevailing conditions in the market for funds and the risk involved in providing prescribed transmission services"*

36. In our view this clearly requires one to take into account what actually happens in the market for funds (in this case, equity markets) not simply what is predicted by a highly stylised model such as the Sharpe CAPM assuming the government bond is a good proxy for the risk free rate. If one has regard to actual evidence from equity markets, no reasonable person could conclude that a naïve application this formula would be superior to the application of an adjusted formula such as is described above.

37. A failure to adopt this recommendation will, other things equal, deny regulated businesses a reasonable opportunity to recover their efficient costs.

1. Introduction

1.1. Terms of reference

38. CEG has been asked to develop an empirical study of the sensitivity of the return on Australian equity to the beta of the equity. We have been asked to specifically test whether the actual sensitivity to beta is equal to the sensitivity implied by the Sharpe (1964) CAPM model and set out at 6A.6.2 (b) of the National Electricity Rules (NER).

39. We have also been asked to explain whether our results are consistent with both:

- empirical results of similar studies in other equity markets around the world; and
- modern asset pricing theory, including developments in the CAPM since Sharpe first outlined his model in 1964.

40. The remainder of this report has the following structure.

i. Section 2 surveys the empirical literature. This section finds that it is a near universal result that zero beta equity earns more than the government bond rate;

ii. Section 3 describes the methodology and results of our empirical study of Australian data. This study confirms evidence from overseas;

iii. Section 4 surveys the theoretical literature for an explanation of the empirical results within the context of the capital asset pricing model (broadly defined); and

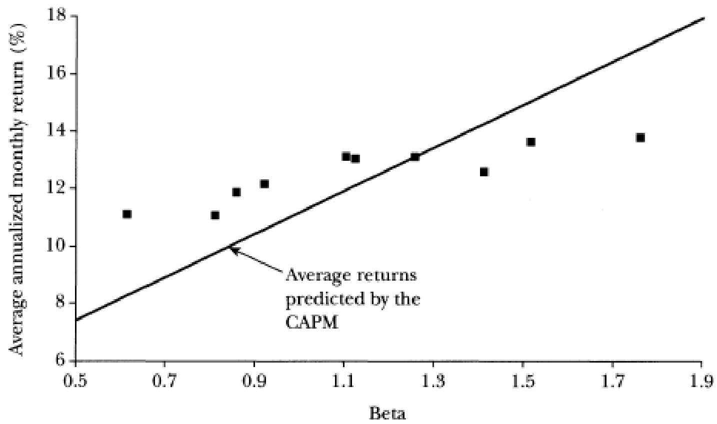
iv. Section 5 provides our recommendations, based on the preceding analysis, for how regulators should interpret observed equity betas when attempting to establish an estimate of the cost of equity for regulated firms.

2. Foreign Empirical Relationships

2.1. The sensitivity of returns to beta

41. The empirical literature unambiguously finds that zero beta equity earns more than the risk free rate and that the sensitivity of equity returns to the value of the equity beta is lower than predicted in the Sharpe CAPM. This general finding is described in the below figure from Fama and French (2004) which demonstrates the difference between the actual relationship between equity beta estimated from market data and equity returns compared to the predicted relationship where the risk free rate is the yield on Government bonds.

Figure 2
Average Annualized Monthly Return versus Beta for Value Weight Portfolios Formed on Prior Beta, 1928–2003



42. In the above graph, the Government bond rate defines the intercept of the CAPM security market line (SML = the dark line). The slope of the line is defined by the market risk premium measured relative to the Government bond rate. That is, the SML as drawn is the SML predicted by the Sharpe CAPM.

43. As is clear from the above graph the actual relationship between beta and market returns is much flatter than that predicted by the CAPM with Government bond yields used as the risk free rate. This is a general finding of the empirical tests of the CAPM as described by Fama and French.

> *"The Sharpe-Lintner CAPM predicts that the portfolios plot along a straight line, with an intercept equal to the risk-free rate, R_f, and a slope equal to the expected excess return on the market, $E(R_M)$- R_f. We use the average one-*

month Treasury bill rate and the average excess CRSP market return for 1928-2003 to estimate the predicted line in Figure 2. Confirming earlier evidence, the relation between beta and average return for the ten portfolios is much flatter than the Sharpe-Lintner CAPM predicts. The returns on the low beta portfolios are too high, and the returns on the high beta portfolios are too low. For example, the predicted return on the portfolio with the lowest beta is 8.3 percent per year; the actual return is 11.1 percent. The predicted return on the portfolio with the highest beta is 16.8 percent per year; the actual is 13.7 percent."

44. The classic empirical investigations of the single factor CAPM models were undertaken by: Fama and Macbeth (1973) and Black, Jensen and Scholes (1972). Fama and Macbeth estimate monthly cross-sectional regressions of stock portfolio risk premiums on estimates of the portfolios' equity betas. That is, all equities in the sample are divided into ten different portfolios according to their beta (from low to high beta portfolios). The returns for each portfolio are then compared with their beta and a regression is performed to assess the relationship between a portfolio's beta and the excess returns (relative to the risk free rate) on that portfolio. This is done for every month in the sample period. If the Sharpe CAPM is true, the estimated regression line should, on average, pass through the origin (ie, zero estimated beta should be associated with zero estimated excess returns).

45. In more technical terms, for each month *t* between 1935 and 1968, the researchers ran a cross-sectional regression of the form:

$$r_{pt} - r_{ft} = \lambda_{0t} + \beta_{pt} \cdot \lambda_{1t} + e_{pt}$$

where r_{pt} denotes the month t return on portfolio p and r_{ft} is the risk-free rate in month t. β_{pt} is the estimated equity beta of portfolio p in month t. The average of the monthly estimated λ_{0t} values is significantly positive and greater than 0.48 percent per month (greater than 5.9% pa). **That is, zero beta equity is predicted to have an annual return of around 6% above the government bond rate**.

46. Similarly, the average of the monthly estimated λ_{1t} is positive but significantly less than the realized average value of the market risk premium. That is, stock returns were estimated to be sensitive to beta but not as sensitive as predicted by the Sharpe CAPM.

47. Fama and Macbeth also test the fundamental CAPM prediction of a positive linear relation between expected risk premiums and equity betas by including both the stock's squared equity beta and the standard deviation of the stock's return as additional explanatory variables in the regressions. Inclusion of a beta

squared term allows a test for linearity. Inclusion of a measure of non-market-related uncertainty allows a test of the Sharpe CAPM prediction that only beta and not standard deviation attracts a risk premium. Fama and Macbeth do not reject the null that the average risk premium is unrelated to both squared betas and non-market risk and hence conclude that they cannot reject the hypothesis that returns are linearly related to beta.

48. Fama and Macbeth conclude from their results that they can reject the Sharpe CAPM (due to the statistically significant positive excess return earned by zero beta stock) but cannot reject the Black CAPM (which predicts that zero beta stock to have positive excess returns relative to government bonds but that there is still a linear relationship between expected stock returns and beta). (As we discuss later, Fischer Black relaxed the assumption by Sharpe that investors can borrow at the same rate as the government. As a consequence of making this assumption more realistic, the Black CAPM predicts the expected return on zero beta equity must be above the government bond rate but that returns will still be linearly related to beta. Hence, the findings by Fama and Macbeth do not reject the Black CAPM.)

49. Using data for the 1931-65 period, Black, Jensen and Scholes (1972) regressed the average monthly returns on 10 portfolios on the portfolios' historical betas. The average monthly market risk premium over the period is 1.42%. The estimated return on zero beta equity in excess of the government bond rate is 0.359% per month, significantly greater than the zero predicted by Sharpe's model. **That is, the return on zero beta equity is estimated to be 4.4% pa above the government bond rate.** Like Fama and Macbeth, Black, Jensen and Scholes conclude that (i) they can reject the Sharpe CAPM and (ii) the data are consistent with the Black CAPM.

50. The conclusion of this literature is that the Sharpe model does not describe reality and will under (over) estimate the cost of equity for low (high) equity beta equity. The Black CAPM was not rejected by these empirical tests.

51. More recent tests find an even flatter relationship between market returns and beta. Fama and French (2004) state:

> *"Fama and French (1992) also confirm the evidence (Reinganum, 1981; Stambaugh, 1982; Lakonishok and Shapiro, 1986) that the relation between average return and beta for common stocks is even flatter after the sample periods used in the early empirical work on the CAPM."*

52. More recently, Campbell and Vuolteenaho (2004) have estimated that the return on zero beta equity is above not only the government bond rate but also is above the market return. That is, lower equity betas are actually associated with higher

returns rather than the opposite as predicted by the single period CAPM models (Sharpe and Black).

53. The guest finance professor at the 2008 ACCC regulatory conference, Professor Jagannathan, also presented results of his own studies that confirmed this relationship. For example, he presents results of a similar study restricting his sample to 'well established firms' in the US in the period 1937 to 2007. This data set covers 75% of the Centre for Research into Security Prices (CRSP) data set. He finds when running an equivalent test to that performed by Fama and Macbeth that the zero beta stock earns 0.63% per month (7.8% annually) above the risk free rate.[7]

2.2. The sensitivity of returns to factors other than beta

54. As we discuss below, more recent empirical tests of the CAPM have rejected the use of any model that has equity beta as the sole determinant of relative risk (this includes the Black CAPM).

55. Key papers in this literature are summarised in the table below. The findings of the literature can be summarised as:

    a. the return on zero beta equity exceeds the return on government bonds (inconsistent with the Sharpe CAPM but consistent with the Black CAPM);

    b. factors other than equity beta (such as firm size and book to market value) explain realised returns (inconsistent with both the Sharpe and Black CAPM); and

    c. when one accounts for these other factors then the role of equity beta becomes insignificant in explaining returns (inconsistent with both the Sharpe and Black CAPM).

56. The most significant of all documented anomalies relative to the single factor CAPM is the consistently higher returns on equity with high book to market values (value stock) relative to equity with low book to market values (growth stock).[8] This is true even after controlling for differences in their CAPM betas. In fact,

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7 See slide 26 at http://www.accc.gov.au/content/item.phtml?itemId=836242&nodeId=d94f0f6388fac76502937d425a2285f7&fn=Ravi%20-%20CAPM%20Debate%20July11th2008.pdf

8 The more market value exceeds the book value of assets the greater is the implied valuation of the growth potential for the firm (ie, the ability to derive value above and beyond that associated with assets already on the books). This is why firms with low book to market values are termed growth stocks and firms with high book to market values are termed growth stocks.

value stocks outperform growth stocks on average, yet value stocks typically have CAPM betas below one and growth stocks typically have CAPM betas above one. To explain this empirical regularity by a model in which differences in expected returns were driven by differences in CAPM betas would require that one use the Black CAPM with a negative slope for the SML; i.e., with a value for the expected return on zero beta equity that exceeds the expected return on the market.

57. Campbell and Vuolteenaho (2004) state:

> *"It is well known that the CAPM fails to describe average realized stock returns since the early 1960s, if a value-weighted equity index is used as a proxy for the market portfolio. In particular, small stocks and value stocks have delivered higher average returns than their betas can justify. Adding insult to injury, stocks with high past betas have had average returns no higher than stocks of the same size with low past betas."*

58. This empirical finding is impossible in the Sharpe CAPM but can be explained by the Merton CAPM, where beta is not the only risk factor. Appendix B summarises the key papers in the empirical literature that have stood the test of time (ie, whose results have shown to be robust both over time and to testing by other academics).

3. Australian Empirical Relationships

59. In this section we replicate the results of Black, Jensen and Scholes (1972) and Fama and Macbeth (1973) using Australian stock data from 1964 to 2007. The purpose for doing this is to test whether the Sharpe CAPM formula is the most accurate predictor of required equity returns in Australia over this period.

3.1. Methodology

60. The main result of the Sharpe CAPM model is that a single factor, beta, determines the expected return of a firm in excess of the risk-free rate, according to the following formula:

$$E(r_i - r_f) = \beta_i \cdot E(r_m - r_{rf})$$

Where: r_i is the return on asset *i*;

r_f is the risk-free rate of interest;

r_m is the return on the value-weighted market portfolio; and

β_i is the 'systematic risk' of asset *i.*

61. A corollary of this formula is that a stock with a zero beta should earn, on average, returns that are equal to the risk-free rate of interest. The rationale behind this result is that any uncertainty inherent in the returns of this stock is not correlated with market returns and hence this uncertainty is completely diversifiable. The Sharpe CAPM implies that stock with a beta of zero will add no risk to a diversified portfolio of stocks and therefore in equilibrium its price will adjust so that its returns to investors will be equal to the risk-free rate of interest.

62. To test the specification of the Sharpe CAPM it is necessary to estimate whether the following equation describes actual returns on each stock *i*:

$$r_{it} - r_{ft} = \lambda_{0it} + \lambda_{1it}\beta_{it} + u_{it} \quad (1)$$

63. That is, having estimated betas for each stock under the assumption that the Sharpe CAPM holds, regress excess returns on the stocks against beta. Under a null hypothesis of the Sharpe CAPM being valid, we would expect to estimate an average value of $\lambda_{0i} = 0$ and an average value of $\lambda_{1i} = r_m - r_{rf}$.

3.1.1. Description of data

64. In order to test the functional specification of the Sharpe CAPM we have used the Share Price and Price Relatives (SPPR) database, supplied by the Centre for Research in Finance of the Australian School of Business, University of New South Wales.

65. This database contains information on monthly returns for 4950 Australian listed stocks dating back to as far as 1958, although the most complete information is available over the period from December 1973 to December 2007. For reasons described below we only use data from January 1964.

66. For each month and each stock listed on the exchange, the database provides a company price relative – a measure of return on investment that is calculated in the following manner:

$$\mathrm{Prel}_t = \frac{\mathrm{Price}_t + \mathrm{Adjust}_t + \mathrm{Dividend}_t}{\mathrm{Price}_{t\text{-}1}} - 1$$

67. As the equation above demonstrates, the price relatives take into account dividends paid by a company in any particular month. Furthermore, changes to a company's capital structure, such as consolidations, splits, repayments of capital and other non-dividend distributions are captured by the capital adjustment term to ensure that price relatives are comparable over time.

68. Table 1 below provides some summary statistics of this dataset.

Table 1: Summary statistics of SPPR dataset

Variable	Obs	Mean	Median	Standard deviation	Period
Stock returns (monthly)	227,486	0.0194	0.0000	0.2196	Jan 1964 – Dec 2007
Market returns (value weighted)	408	0.0127	0.0149	0.0492	Jan 1974 - Dec 2007
Market returns (equal weighted)	408	0.0218	0.0228	0.0567	Jan 1974 - Dec 2007
Risk-free rate	408	0.0069	0.0061	0.0029	Jan 1974 - Dec 2007

69. One result from Table 1 that is of interest is the difference between the value-weighted return on the market and the equal-weighted return on the market. The

average return on the market over the 34 years until the end of 2007 was, 1.27% per month, equivalent to an annual rate of 16.35% or approximately 7.75% in excess of the risk-free rate. However, the average return on the equal-weighted market over the same period was 2.18% per month, equivalent to an annual rate of 29.54%, or 20.94% in excess of the risk-free rate.

70. In other words, smaller firms, on average, earn higher returns than larger firms. This is consistent with the findings of Banz (1981) and the findings of other authors as described in the summary table at the end of the last section. Exactly why this is the case is not fully understood in the profession.

3.1.2. Regression methodology

71. In order to test the validity of equation (1) we prefer to use data for a large number stocks and all available time periods to estimate λ_0 and λ_1, namely the intercept (λ_0 = zero beta excess return) and slope (λ_1 =market risk premium relative to zero beta return) in a regression of excess returns on betas. The methodology follows that first applied by Black, Jensen and Scholes[9] and subsequently by Fama and MacBeth.[10] We describe this methodology and the motivation for it in the following paragraphs.

72. For each month, *t*, from January 1974 to December 2007, betas for each stock *i* are estimated separately over the months *t-120* to *t-61* and *t-60* to *t-1*. That is, for a stock in January 1974, two betas are estimated using data from January 1964 to December 1969 and January 1970 to December 1974. Each pair of beta estimates is retained only if it is based on at least 40 observations in each of the five-year sample periods, and the stock has a valid observed return in month *t*. This creates a pair of betas in each month between January 1974 and December 2007 inclusive for each stock that satisfies these conditions in that particular month.

73. The betas above are generated using the following regression:[11]

$$r_{i\tau} = \alpha_{it} + \beta_{it} r_{m\tau} + \epsilon_{it\tau} \qquad (2)$$

[9] Black, F., Jensen, M. and Scholes, M. (1972) The Capital Asset Price Model: Some Empirical Tests, in Jensen, M. (ed.), *Studies in the Theory of Capital Markets*, pp. 79-121.

[10] Fama, E. And MacBeth, J. (1973) Risk, Return and Equilibrium: Empirical Tests, *The Journal of Political Economy*, 81, 3, pp. 607-636.

[11] A alternate regression to estimate beta would be $r_{i\tau} - r_{f\tau} = \beta_{it}(r_{m\tau} - r_{f\tau}) + \varepsilon_{i\tau}$. The risk-free rate is not available from the SPPR dataset before December 1973, so this equation cannot be estimated. The estimate of beta from equation (2) should nonetheless be unbiased.

where: $(t-120) \leq \tau \leq (t-61)$; and $(t-60) \leq \tau \leq (t-1)$.

74. For each time period *t*, ten portfolios of stocks are created by ranking the betas estimated over the time periods from *t-120* to *t-61*. The stocks with the bottom 10 percent of betas are allocated to portfolio 1, the next 10 percent of betas are allocated to portfolio 2, and so on.

75. We then estimate the following regression:

$$r_{pt} - r_{ft} = \lambda_{0t} + \lambda_{1t}\beta_{pt} + \varepsilon_{pt}, \qquad p = 1,2,\ldots,10 \tag{3}$$

where: r_{pt} is the average return of stocks in portfolio *p* at time *t*;

β_{pt} is the average betas of stocks in portfolio *p* at time *t,* where the beta is calculated over the period *t-60* to *t-1*.

76. Estimating equation (3) at each time *t* gives us a time series of 408 estimates for λ_{0t} and λ_{1t}. The simplest way to conduct inference on these values is to take the mean and standard error of the mean and test against the null hypotheses that:

i. $\lambda_0 = 0$ That is, that the average excess return on a stock with a beta of zero is also zero; and

ii. $\lambda_1 > 0$. That is, average returns increase as a stock's beta increases.

Box 1: Why use older betas to construct portfolios?

The rationale for using betas estimated over *t-120* to *t-61* to construct the portfolios and betas estimated over *t-60* to *t-1* to test the specification of the CAPM is well described by Black, Jensen and Scholes.

If we were to construct the portfolios using the same betas that were also used in the regression above, this would tend to introduce a systematic bias into our results. Specifically, those betas that were underestimated would be more likely than otherwise to enter the lower portfolios, whilst those that were overestimated would be more likely than otherwise to enter the higher portfolios. In aggregate, errors of this type will cause a negative bias in the average beta for the lower portfolios and a positive bias in the average beta for the higher portfolios.

The biases described above would tend to lead to biases in the estimated coefficients and invalidate statistical inference on these estimators. In this case, the problem may be alleviated by using an instrumental variable to proxy for the estimate of beta over t-60 to t-1 in selecting the stocks to be allocated to each portfolio to be used in the regression. It is reasonable to use an historical estimate of beta for the same stock to select the stock to be allocated to these portfolios. Whatever measurement error enters the construction of these portfolios will not be systematically correlated with the measurement error of the betas within the portfolio, and hence the bias is eliminated.

77. A further, and more conservative, method for statistical inference is to regress the time series for γ_0 and γ_1 against the excess return on the market, in the following regressions:

$$\lambda_{0t} = \gamma_0 + \gamma_1(r_m - r_f) + \omega_t \tag{4}$$

$$\lambda_{1t} = \theta_0 + \theta_1(r_m - r_f) + \xi_t \tag{5}$$

78. The effect of these regressions is to remove from the estimates of λ_0 and λ_1 any correlations between these estimates and the market return that may exist across time, which cannot be controlled for in a cross-sectional regression. Under the assumption that the Sharpe CAPM is valid, the null hypotheses to test are $\gamma_0 = 0$, $\gamma_1 = 0$, and $\theta_0 = 0$ and $\theta_1 = 1$. In other words, if $\gamma_0 = 0$ and $\gamma_1 = 0$ then the return on a zero beta equity (λ_{0t}) will also equal zero – which is a prediction of the Sharpe CAPM. Similarly, if $\theta_0 = 0$ and $\theta_1 = 1$ then the sensitivity of stock returns to beta (λ_{1t}) will be equal to $(r_m - r_f)$ - which is also a prediction of the Sharpe CAPM.

3.2. Regression results

79. The results for estimating equations (3), (4) and (5) over the period January 1974 to December 2007 are set out in Table 2 below.

Table 2: Summary of regression statistics – all firms equal weighted

Parameter	Coefficient (monthly)	Annualised	Standard error	t-value*	p-value*
λ_0 (average)	0.005859	7.26%	0.002068	2.833	0.005
λ_1 (average)	0.009300	11.75%	0.003682	2.526	0.012
γ_0	0.005018	6.19%	0.002052	2.445	0.015
γ_1	0.1476	n.a.	0.0414	3.563	0.000
θ_0	0.004537	5.58%	0.003090	1.468	0.143
θ_1	0.8362	n.a.	0.0624	-2.627	0.090

* *t-value and p-value measured against a null hypothesis of zero, except for θ_1, which is measured against a null hypothesis of one.*

80. The average coefficients estimated from regression (3) are both positive and statistically significant. The results suggest that a stock with zero beta earns, on average, 7.26% per annum above the risk-free rate and that exposure to the average level of systematic risk is valued at a further 11.75% per annum. The p-values attached to λ_0 is 0.005 which suggests that we can be 99.5% confident that the true value of λ_0 is greater than zero (ie, that zero beta equity earn more than the government bond rate). Similarly, the p value for λ_1 suggests that we can be 98.8% confident that beta plays some role in determining returns within our sample.

81. We note that, the regression of λ_0 against the excess return on the market (relation (4)) also suggests a significant and positive intercept, with an expected return on a zero beta stock of 6.19% per annum above the risk-free rate.

82. It is interesting to note that given the results of equation (3), a firm with a beta of one would, on average, expect to earn 19.01% per annum above the risk-free rate (being the sum of λ_0 and λ_1). This is much higher than the excess return on the value-weighted market, of 7.75%. This occurs because equation (3) seeks to link the expected excess return on any stock to its beta. But a typical stock will also be a relatively small stock. (The Australian market is highly concentrated with just 10 stocks comprising approximately ½ the market value of the entire set of ASX-listed stocks.) As previously stated at paragraph 69 the average excess

return on the equal-weighted market is considerably higher than the average excess return on the value-weighted market.

83. The relation between firm size and average returns cannot be explained by the CAPM. It is not the case that small stocks typically have much higher betas than large stocks. The regression underlying equation (3) must by definition pass through the point corresponding to (a) the average of the excess returns on the 10 portfolios and (b) the average of the betas of the 10 portfolios. The average excess return on the 10 equal-weighted portfolios is simply the equal-weighted market excess return. The average of the betas of the 10 equal-weighted portfolios is very close to 1.[12] Thus, the estimated slope in relation (3) is approximately the difference between the excess return on the equal-weighted market and the estimated intercept; i.e., approximately 20.94% − 8.6% = 12.34%.

84. In light of the results of the regression at Table 2, we consider it a useful exercise to examine the effect on the results of excluding many of the smaller firms in the data. Since it is apparent that the smaller firms in the population, that tend to have the highest returns, are driving the results of the regression, running the same equation using betas and returns derived only from larger firms will allow us to observe whether a similar relationship exists for these firms considered independently.

85. Table 3 below show the results of the same regressions conducted on stocks that are amongst the 300 with the greatest capitalisation on the stock exchange in the month immediately preceding the measured return. The number of 300 has been chosen because it is common to refer to an index of the 300 largest firms on the ASX.[13] However, ultimately it is an arbitrary measure of what constitutes a 'large' firm, and we provide further results under different assumptions.

12 If in fact small stocks did have very high betas and their returns could be explained by the Sharpe CAPM then, given a 6% MRP, the beta of the equal-weighted market would have to equal 3.49; i.e., 20.94% ÷ 6%.

13 In order to more accurately estimate stock betas from a five-year window of monthly return data, a criterion was imposed that the stock has at least 40 monthly return observations in each five year period.. Not all the 300 firms satisfy this criterion and hence less than 300 firms contribute to the estimates in Table 3 in any particular month.

Table 3: Summary of regression statistics for 300 largest firms

Parameter	Coefficient (monthly)	Coefficient (annual)	Standard error	t-value*	p-value*
λ_0	0.006548	8.15%	0.002222	2.947	0.003
λ_1	0.000616	0.74%	0.003117	0.198	0.844
γ_0	0.005663	7.01%	0.002206	2.567	0.010
γ_1	0.1553	n.a.	0.0445	3.486	0.000
θ_0	-0.004032	-4.73%	0.002418	-1.667	0.096
θ_1	0.8159	n.a.	0.0488	-3.770	0.000

* *t-value and p-value measured against a null hypothesis of zero, except for θ_1, which is measured against one.*

86. The results in Table 3 indicate that the returns of the 300 largest firms at each month are broadly consistent with those of the value-weighted market. A firm with a beta of one would, under the estimates above, expect to earn 8.89% above the risk-free rate – a similar differential to the excess return on the value-weighted market of 7.75%.

87. When we reduce our sample to only cover the 300 largest firms, it remains the case that the return on a zero beta stock is highly statistically significantly different from zero (we can be 99.7% confident that λ_0 is more than zero). However, this is no longer true of the sensitivity of equity returns to beta. We can now only be 15.6% confident that beta plays any role in determining excess returns (ie, that γ_0 is different from zero).

88. Based on these results, the only reasonable conclusion is that, amongst large firms, beta plays a relatively small role in explaining the returns of a stock. Similar results are obtained for estimates of these parameters for the 200, 100, and 50 largest firms, in Table 4 below.

Table 4: Summary of regression statistics for large firms

Number of firms	Parameter	Coefficient (monthly)	Coefficient (annual)	Standard error	t-value	p-value
300	λ_0	0.006548	8.15%	0.002222	2.947	0.003
	λ_1	0.000616	0.74%	0.003117	0.198	0.844
200	λ_0	0.005816	7.21%	0.002547	2.283	0.023
	λ_1	0.001020	1.23%	0.003224	0.316	0.752
100	λ_0	0.008210	10.31%	0.003371	2.436	0.015
	λ_1	-0.001781	-2.12%	0.003712	-0.480	0.632
50	λ_0	0.006874	8.57%	0.003786	1.816	0.070
	λ_1	-0.000259	-0.31%	0.003873	-0.067	0.957

89. These results tend to mirror the results with the largest 300 firms. Specifically, the slope coefficient against beta remains insignificant, whilst the intercept is positive and significant at the 97.5% confidence level in each case except for 50 firms (where it is significant at the 93% confidence level). This is likely because a sample of 50 (or less firms) is likely to generate results with quite high standard errors. Indeed, Table 4 shows the standard error of estimates consistently increasing as the number of firms included in the sample decreases.

90. Finally, in order to confirm the results above, we have conducted a regression where the portfolios are constructed using the average betas and returns, weighted by the size of each firm. This regression is not truly value-weighted, since we do not weight each portfolio by the aggregate size of the firms contained within it. However, it limits the extent to which small firms can influence the results of any one portfolio. The results of this regression are shown in Table 5 below.

Table 5: Summary of regression statistics for value-weighted portfolios

Parameter	Coefficient (monthly)	Coefficient (annual)	Standard error	t-value*	p-value*
λ_0	0.006375	7.92%	0.002577	2.474	0.014
λ_1	0.000315	0.38%	0.003334	0.094	0.925
γ_0	0.005289	6.53%	0.002555	2.070	0.039
γ_1	0.1907	n.a.	0.0516	3.697	0.000
θ_0	-0.004267	-5.00%	0.002717	-1.571	0.117
θ_1	0.8045	n.a.	0.0548	-3.565	0.000

* *t-value and p-value measured against a null hypothesis of zero, except for θ_1, which is measured against one.*

91. The results of this regression are consistent with those using only the largest firms. The intercept is significant (at 98.6%) and positive, at 7.92% per annum, whereas the slope is insignificantly different from zero.

92. It is interesting to question what is causing the significant slope coefficient measured from equation 3 when all firms (including low value stocks) are included in the regression. *A priori*, there would appear to be two possible causes for this discrepancy, being:

i. beta has no impact on the returns of large firms, but is determinative of the returns of small firms; or

ii. beta is not determinative of the return of any firm and the slope estimated from equation (3) was generated because small firms have higher returns than large firms due to their greater exposure to non-beta risks (e.g. the risks recognized by the Merton CAPM discussed in section 4.1.2) and the degree of such exposures is correlated with beta for small firms.

93. It appears that the first explanation is best borne out by the data. That is, when we perform a regression restricted to the smallest 500 firms we estimate both a statistically significant intercept (excess return relative to the risk free rate for zero beta equity) and a statistically significant relationship between beta and equity returns.

3.3. Regression results - graphical

94. In this section we show graphically our empirical results. In each figure we show ten data points that represent the average beta and the average return for each

of our ten portfolios across all 408 monthly observations. The fitted line between these data points is shown in each graph (blue line).[14] This is contrasted with the predictions of the Sharpe CAPM (red line).

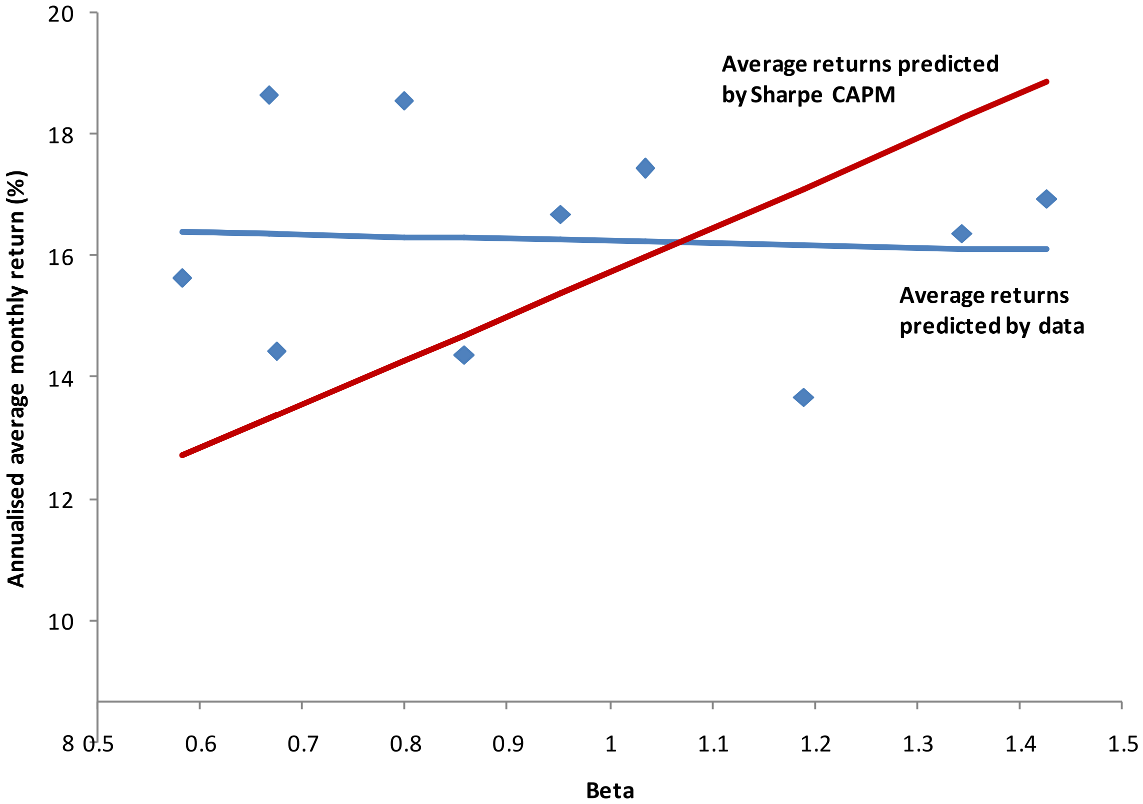
95. As can be seen, in all cases the estimated return on zero beta equity is above the risk free rate (ie, the intercept of the fitted line is above the return on government bonds). As was described in the previous section, this result is statistically significant in all of the regressions illustrated graphically below. A consequence of this is that expected returns on stocks with beta of less than one are higher than the return predicted by the Sharpe CAPM.

96. All of these graphs compare the average return for each portfolio predicted by the Sharpe CAPM assuming that the risk free rate is the government bond rate (a line joining two points being the average government bond rate and zero beta and the average market return and a beta of 1.0) and the return predicted by the data (being a fitted line through the ten portfolios). The most important point of comparison between these two lines is their slope. This difference demonstrates the difference between the actual sensitivity of returns to beta and the sensitivity predicted by the Sharpe CAPM.

97. Less relevance can be attached to the ‘level’ of the predicted returns predicted by the data. This tends to be higher than the Sharpe CAPM predictions (intersecting at a beta materially greater than 1.0 in all but Figure 2). However, this reflects the fact that (in all but Figure 2) small firms are given the same weight as large firms and small firms tend to have higher returns than large firms. As a consequence the average returns predicted by the data tends to be higher than the average returns predicted by the Sharpe CAPM because the former gives greater weight to high performing small firms (except for in Figure 2).

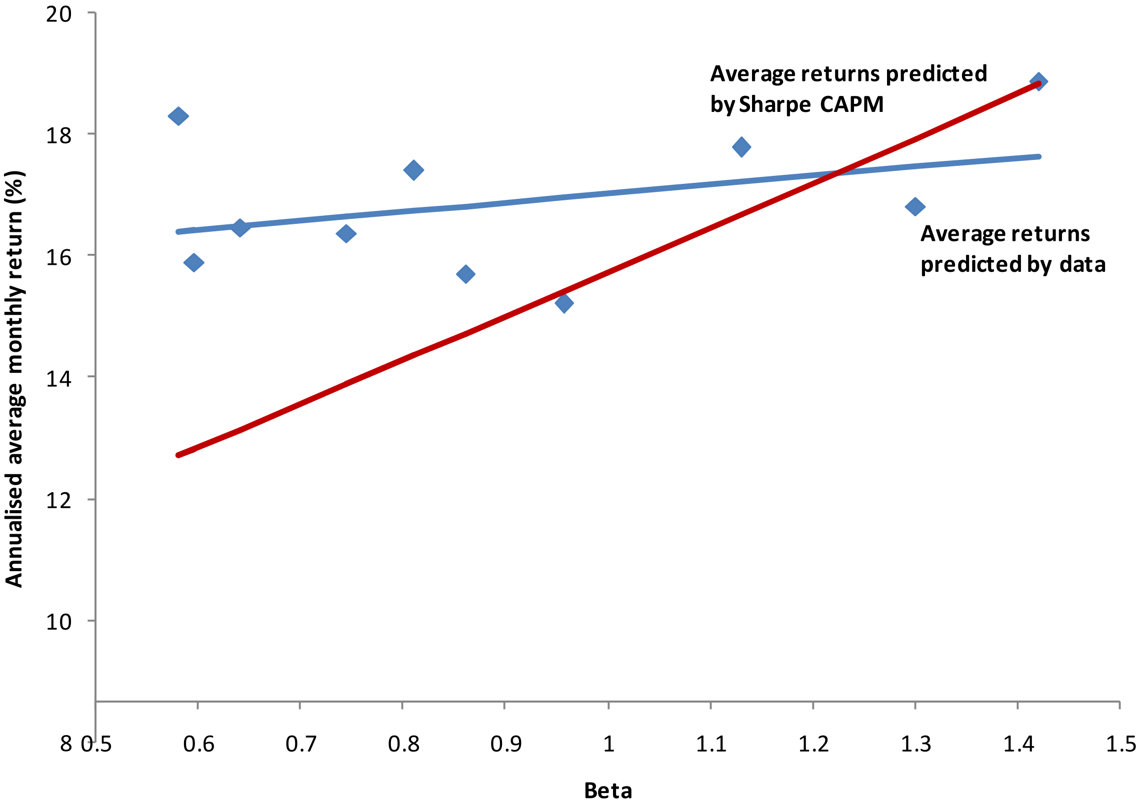
[14] This represents the fitted line derived using the Black-Jensen-Scholes methodology (which is slightly different to the Fama-MacBeth methodology, but gives very similar results)

Figure 2: All stocks included in sample (value weighting to each stock)



98. This figure demonstrates graphically the results described in Table 5. It shows that when portfolio betas/returns are based on the average after weighting each firm's beta/return by firm size there is, if anything, a negative relationship between beta and returns. However, zero beta stock still earn excess returns well above the zero rate predicted by the Sharpe CAPM.

Figure 3: Sample restricted to 300 largest stocks (equal weighting to each stock)



99. This figure illustrates graphically the results described in Table 3. This figure demonstrates that when portfolio betas/returns are based on the equal weighted average of each of the three hundred largest stocks' beta/return there is a slightly positive relationship between beta and returns. However, zero beta stock still earn excess returns well above the zero rate predicted by the Sharpe CAPM.

100. The fitted lines in Figure 2 and Figure 3 can be thought of as representing:

i. the relationship between the return on a stock and its beta if the stock was selected at random with all stocks having an equal probability of selection irrespective of the size of the company (Figure 3); versus

ii. the relationship one would expect if the probability of selecting a stock was proportional to that stock's market weighting. (Figure 2).

101. The following three figures demonstrate graphically the results described in Table 3. It can be seen that the slope of the fitted line (sensitivity to beta) falls as the sample is restricted to larger firms.

102. However, zero beta stock still earn excess returns well above the zero rate predicted by the Sharpe CAPM in all three regressions.

Figure 4: Sample restricted to 200 largest stocks (equal weighting to each stock)

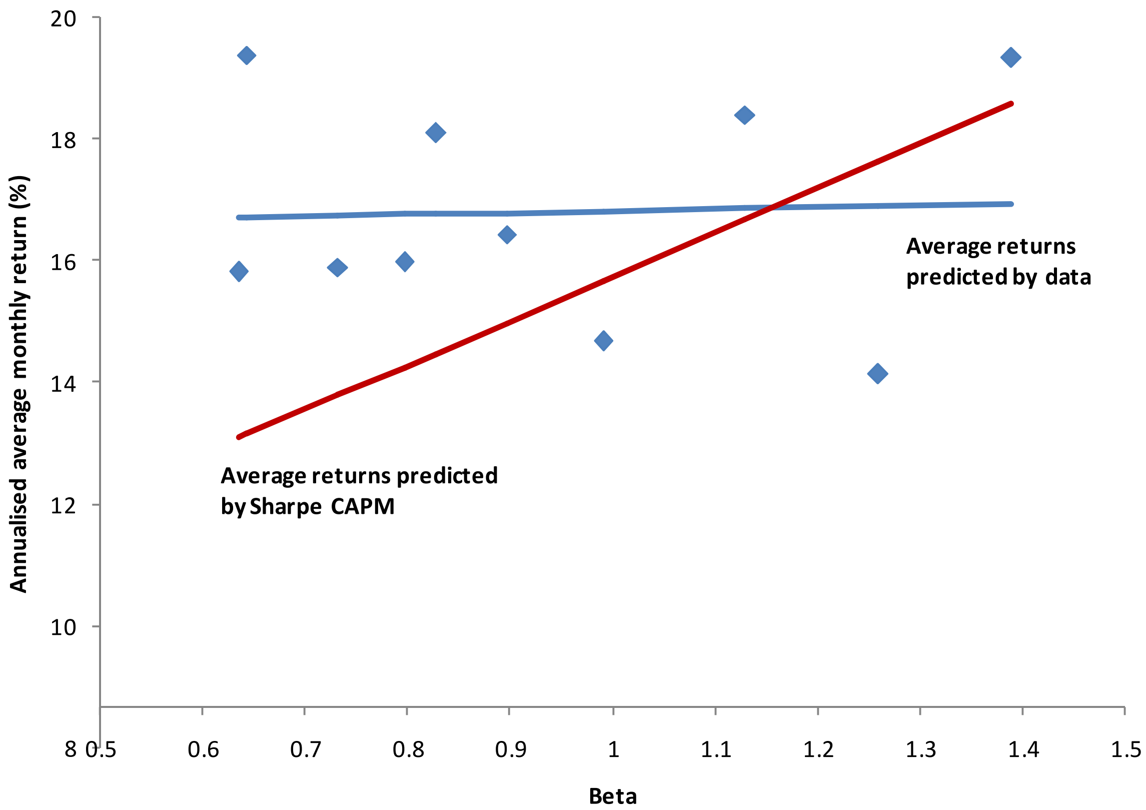


Figure 5: Sample restricted to 100 largest stocks (equal weighting to each stock)

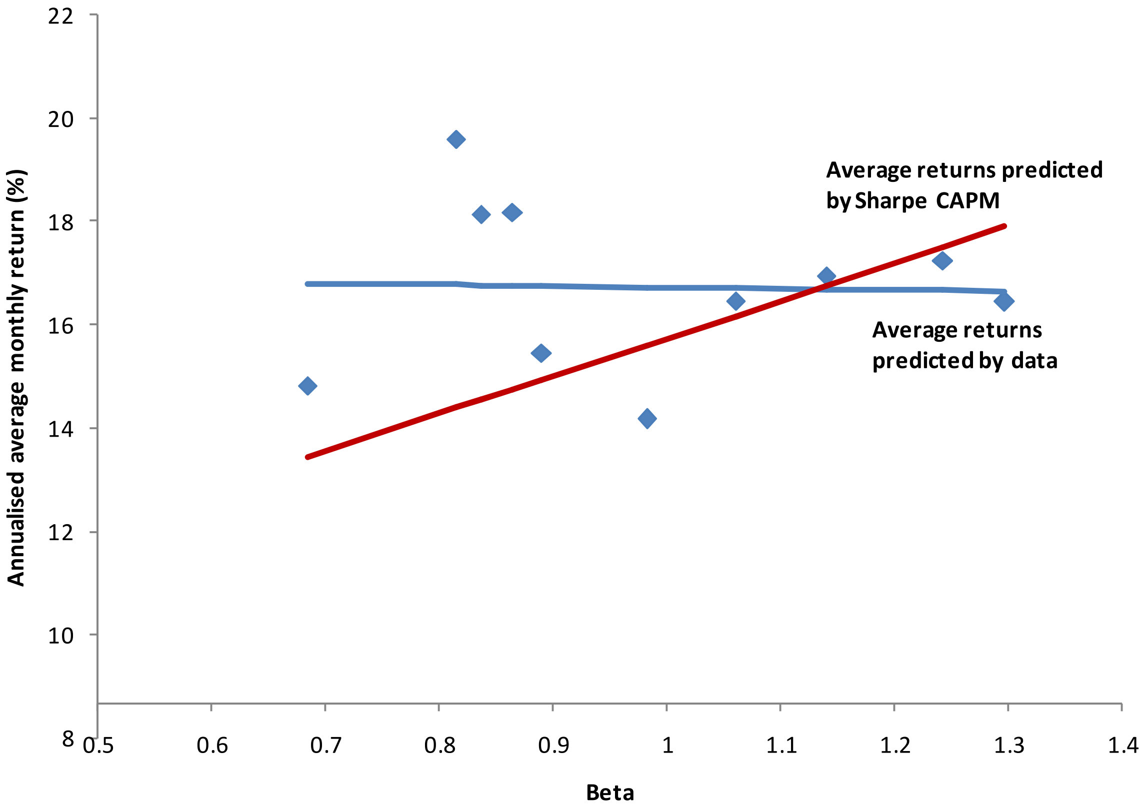
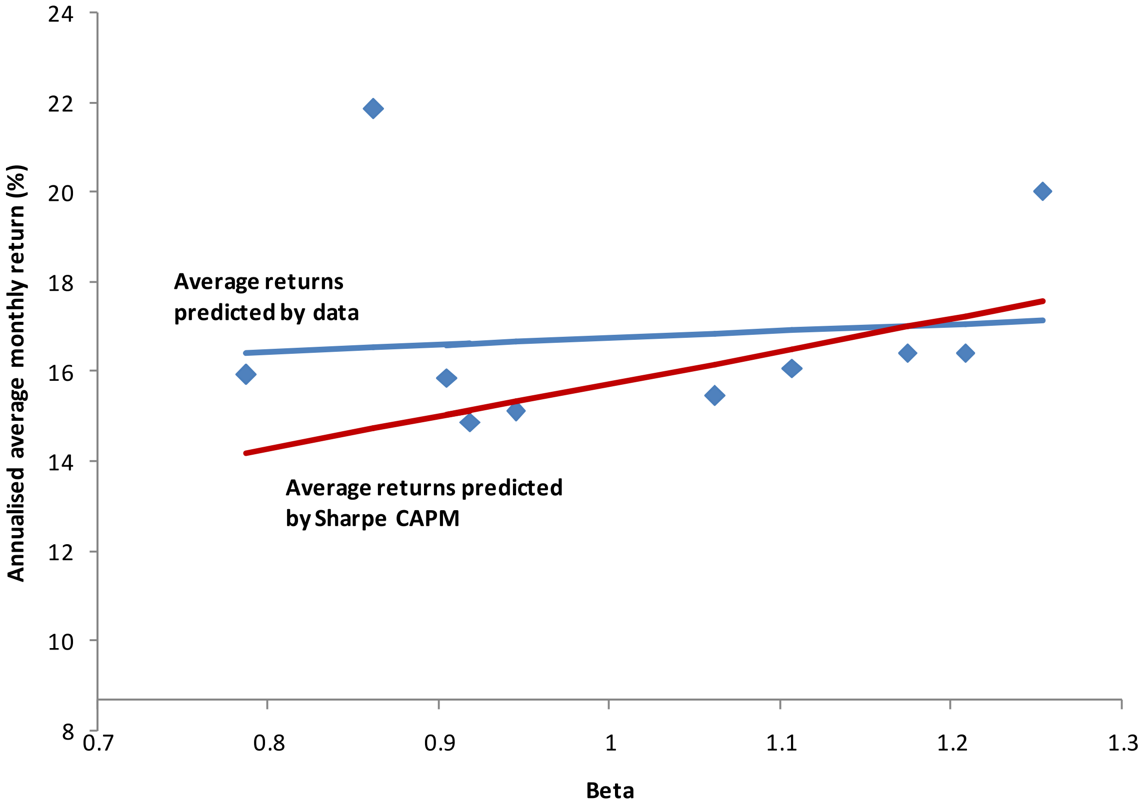
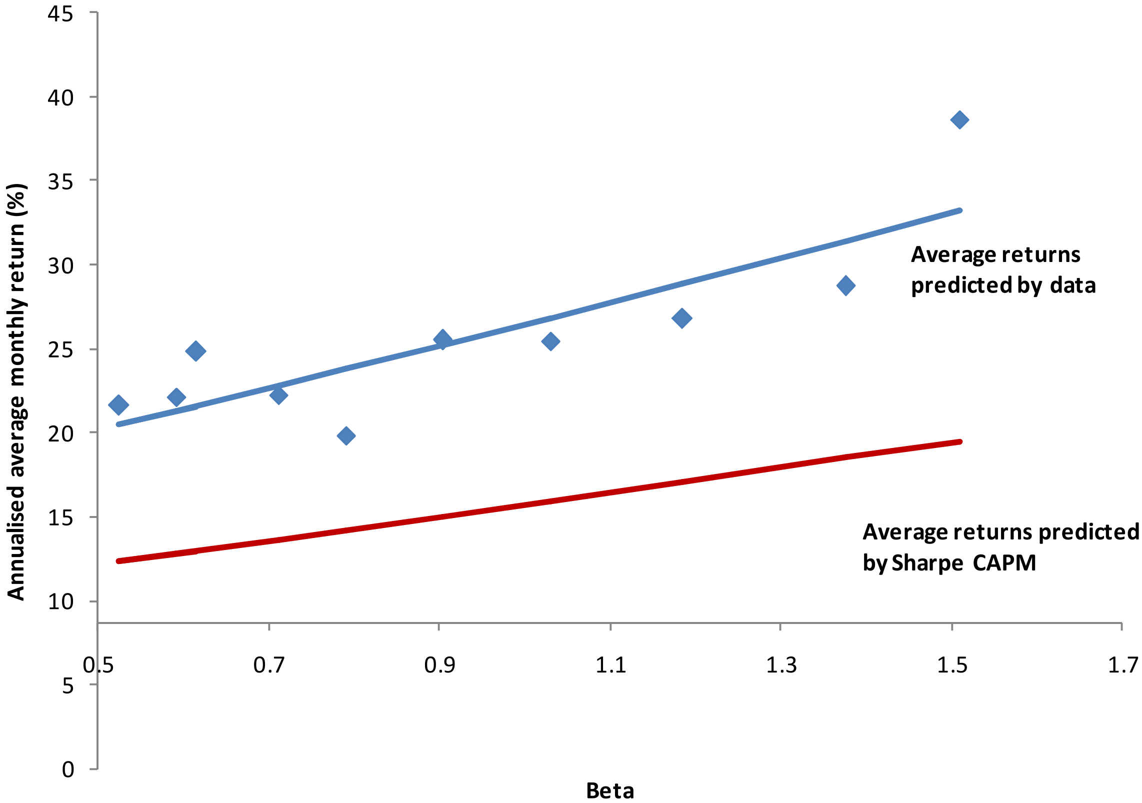


Figure 6: Sample restricted to 50 largest stocks (equal weighting to each stock)



103. The below figure illustrates graphically the results described in Table 2. It demonstrates the fact that small firms tend to earn significantly higher returns than predicted by the Sharpe CAPM and than other firms with the same betas.

Figure 7: All stocks included in sample (equal weighting to each stock)



3.4. ESCV discussion in the Victorian GAAR

104. CEG (then CECG) has presented the international evidence (which is largely the same as the Australian evidence described above) to the Essential Services Commission of Victoria (ESCV) in the context of the Victorian gas access arrangement review (GAAR). In its final decision the ESCV

- did not dispute our conclusions;

- describes our report as "comprehensive";[15]

- acknowledges that the Sharpe CAPM has "come in for considerable criticism in recent years";[16]

105. However, the Commission argue that it was constrained not to have regard to our recommendations on the basis of regulatory precedent and legalistic grounds.[17] When it was argued that the Commission was not so constrained the Commission in its Further Final Decision made the following statement.

> *"Having considered the evidence before it, the Commission acknowledges that it is possible that for low beta stocks the (Sharpe) CAPM may not be the best predictor of returns for firms with a beta other than 1.0.*
>
> *"However, the Commission is not satisfied that it is positively the case that the (Sharpe) CAPM is not a good predictor of returns for firms with a beta less than 1.0. Further, the Commission is not satisfied as to the extent to which the (Sharpe) CAPM may not accurately predict return for firms with a beta less than 1.0, such that it could make an adjustment for this issue. Even if the Commission was satisfied that some adjustment should be made as a result of estimation bias in the application of the Sharpe CAPM, there was not sufficient material before the Commission that would permit it to assess the magnitude of any such adjustment."* (Page 35)

106. Importantly, the ESCV has not rejected our position. The ESCV has simply stated that "*it is not satisfied that it is positively the case that the (Sharpe) CAPM is not a good predictor of returns for firms with a beta less than 1.0".* The logic expressed in this statement implies that:

   i. unless one is '*positive'* the Sharpe CAPM is biased then one should ignore evidence that suggests that it is more than likely biased (including evidence where there are high levels of statistical certainty that bias exists and where the tests have been performed by leading finance academics including Nobel Prize winners); and

   ii. unless one can be confident about the magnitude of the required adjustment one should make no adjustment.

[15] Second last full paragraph on page 473 of the final decision.

[16] Third last full paragraph on page 473 of the final decision.

[17] See last full paragraph on page 474 of the final decision.

107. Both of these positions are clearly wrong. One should take into account any evidence of bias when making a decision. Doing so will improve the accuracy of the estimation even when the evidence of bias is relatively weak (ie, weak but positive evidence of bias suggests small but positive adjustments will improve accuracy). The stronger the evidence of bias the larger the required adjustment.

108. The second position is equally illogical. If one knows that 'X' is the wrong answer and that the correct answer is between 2X and 3X one does not persist in adopting an estimate of X just because one can't know for sure what the correct estimate is.

3.4.1. ACG advice to the ESCV

109. The ESCV also sought the advice of ACG who similarly did not dispute our findings. Like the ESCV, ACG simply stated that we had not "*conclusively* established" that the Sharpe CAPM results "in *significant*" error.

> *"It is our view that there are potential deficiencies in the matters raised and arguments presented in the CECG submissions. While we accept as a generality that the Sharpe CAPM has been found in many instances to underestimate the cost of equity, we do not consider that the arguments and evidence presented in CECG's submissions conclusively establish that use of the Sharpe CAPM results in significant underestimation of the cost of equity for regulated utility businesses such as the gas distribution businesses.*
>
> *In summary, our reasons for this view are as follows.*
>
> - *Empirical studies that test the predictive power of the Sharpe CAPM have mixed results and may not support the view that the Sharpe CAPM would necessarily under-estimate the cost of equity for businesses with the particular characteristics of the gas distribution businesses. We are not satisfied that CECG has undertaken a full and balanced review of relevant empirical studies, and we suggest that studies not addressed by CECG would support contrary findings (i.e., that the Sharpe CAPM does not systematically under-estimate the cost of equity for businesses with low equity betas).*
>
> - *Furthermore, we consider that a more detailed study of businesses with characteristics of the gas distribution businesses would be necessary to reach a definitive view on whether the Sharpe CAPM would systematically under-estimate costs of equity for businesses of this type. The theoretical basis for considering that the Sharpe CAPM would underestimate the cost of equity for businesses such as the distribution businesses has not been conclusively established. It is our view that CECG has not established that*

the theoretical basis for the underestimation of the cost of equity (a high level of 'theta risk' or covariance of returns of a business with reductions in investment opportunities) is necessarily of significance for businesses with characteristics of the gas distribution businesses. There are reasons for considering that this may not be the case. Again, further analysis would be necessary to reach a definitive view on this issue."

110. The ACG did not point to any 'actual deficiencies' in our work but only raised the prospect of 'potential deficiencies'. Similarly, the ACG do not reference any studies that we have omitted and that would change our conclusions but rather state that they are *'not satisfied'* that we had undertaken a full and balanced review of relevant empirical studies.

111. The ACG repeat the argument that we have not '*conclusively'* established bias in the Sharpe CAPM. In summary, it would appear that ACG has raised two issues for further consideration in relation to this work:

    i. Whether or not there are other studies that have overturned the empirical findings we reported in 2007;

    ii. Whether a theoretical basis exists for believing that these findings would apply equally to regulated energy businesses;

112. In this work we have extended the empirical work reported previously by virtue of undertaking a study with Australian data – which arrived at the same conclusion as the studies in other countries we have cited. We have also extended the theoretical discussion describing why we would expect the sensitivity of required returns to beta to be no different for regulated utilities than for the average of other firms in the economy.

113. We would expect that this would now establish with a sufficient degree of certainty the relevance to Australian regulators of the results we have previously reported

114. In this regard, we note that the ACG and ESCV have stated that they were 'not satisfied' that we had 'conclusively' or 'definitively' or 'positively' that the Sharpe CAPM would underestimate returns for utilities with beta less than 1.0. Ignoring the probable existence of bias on the basis that it was not conclusive/definitively/positively proven will, other things equal, deny regulated businesses a reasonable opportunity to recover their efficient costs.

115. For example, it would be wrong to reject our empirical findings on the basis that they only prove that the Sharpe CAPM fails when explaining equity returns in general but does not ‘conclusively’ prove that it fails for utilities.

116. In our view this is also illogical. Any reasonable approach would require that:

i. if it can be shown that the Sharpe CAPM is biased at the level of the aggregate market;

ii. regulators should adjust their application of the Sharpe CAPM to ensure that it better explains utility returns.

117. A relevant analogy relates to the link between smoking and lung cancer. It is well known that, at the level of the general population (where it possible to gather sufficient data), smoking increases the risk of lung cancer. It will be difficult (if not impossible) to prove that what is true for the population will also be true for the individual. It may well be that an individual’s genetic make-up protects them from lung-cancer and makes smoking a low risk activity for them. However, without strong evidence to this effect ignoring the evidence from the population would be folly of the worst order.

4. Theoretical interpretation

118. In this section we examine the theoretical explanations developed in the finance literature to explain the empirical regularity described in the previous two sections, ie, to explain why low beta stock earn on average more than predicted by the Sharpe CAPM.

4.1. Survey of the literature

119. We start by examining the theoretical literature on the CAPM and surveying the advances made since 1964 when the Sharpe CAPM was first introduced.

120. Most theoretical models of risk and reward in the finance literature have built on the foundation laid by the Sharpe CAPM. However, the Sharpe CAPM has some extremely limiting assumptions. The advantage of these assumptions is that they enabled Sharpe to derive a very simple mathematical measure of relative risk (the covariance of an asset's return with the market relative to that of the variance of the return on the market). The disadvantage of these simplifying assumptions is that, as the empirical literature has established, the Sharpe CAPM provides a poor explanation in practice of average realized returns on stock in excess of government bond rates.

121. Of course, all theoretical models must simplify reality in order to focus on the key determinants of risk. The challenge is to design a model where the simplifications in the assumptions do not abstract from issues of importance to the phenomenon under examination. The Sharpe CAPM did precisely this and simplified reality sufficiently that the only determinant of risk was covariance relative to the market portfolio. Other researchers have relaxed the restrictive assumptions of the Sharpe CAPM and, in doing so, have identified other important determinants of expected returns and relative risk. Two of the most unrealistic assumptions of the Sharpe CAPM that have been relaxed are that :

   a. investors can borrow at the government bond rate; and

   b. investors invest for a single period and, on the last day of that period, they consume their entire wealth.

122. We summarise the theoretical literature that examines measures of risk and the determinants of expected returns in models that recognize that investors cannot borrow at CGS rates and that investors view risk and return in the context of their lifetime consumption and savings decisions.

4.1.1. The Black CAPM – only beta matters

123. In the Sharpe CAPM, it is assumed that investors can borrow at the same rate as the Government. That is, it assumed that lenders are just as willing to lend to the Government as to investors wishing to gear their exposure to the risky stock market. This assumption is clearly not grounded in reality.[18] In 1972 Black established the implications of relaxing this unrealistic assumption.

124. When investors cannot borrow at the risk-free rate, then expected returns are given by the 'Black CAPM'. In the Black CAPM, the return on zero beta equity is greater than the yield on government bonds. That is, once it is recognised that investors cannot borrow at the Government bond rate it axiomatically follows that the risk-free (i.e., zero beta) rate used to price equities must be above the government bond rate. The risk-free rate used to price equities is referred to as the zero beta rate since it is the expected return on a stock with zero beta risk.

125. The immediate implication of this is that the relationship between equity beta and risk adjusted return is weaker than in the Sharpe CAPM. In the terminology of these versions of the CAPM the security market line (SML) has a smaller slope and a higher intercept. This is illustrated in the figure below.

[18] The apparently odd assumption that investors can borrow unlimited amounts at the risk-free rate can be rationalized within the derivation of the Sharpe CAPM since stock returns are assumed to be normally distributed. When stock returns are normally distributed, the value of the stock belonging to an investor who holds the market portfolio can turn out to be negative, say -$50. If that investor owes $100, she simply takes the $100 from her pocketbook, repays the loan in full and then has -$150. As with all models, the Sharpe CAPM cannot be judged by the realism of its assumptions alone. Rather, the empirical fit of the model to the data should guide the choice between the Black CAPM and the Sharpe CAPM.

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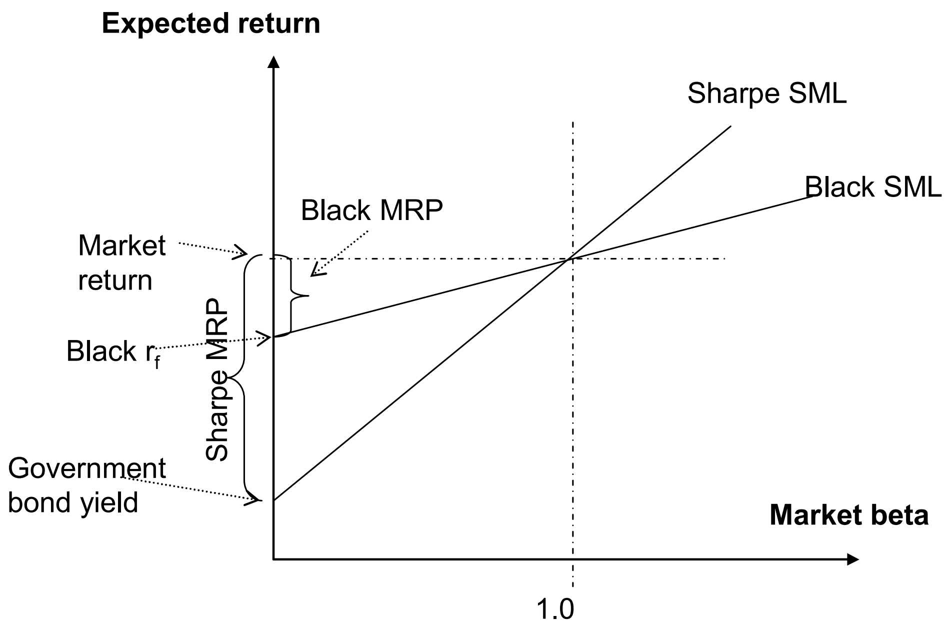

126. Black's key insight was that if one recognizes that investors borrow at rates above the risk-free rate then, even if equity beta is the sole risk that is rewarded in the market place, expected returns on zero beta stocks exceed the government bond rate. That is, stocks with low betas earn more than predicted by the Sharpe CAPM (and *vice versa*).

127. The Black CAPM better explains the empirical literature discussed above – where that empirical literature shows a much lower slope and higher intercept for the SML than is implied by the Sharpe CAPM. We describe how this can be operationalised by the AER in the recommendations to this report.

4.1.2. The Merton CAPM – beta not the only risk factor

128. In 1973 Robert Merton relaxed possibly the most restrictive assumption of the Sharpe CAPM, namely, the assumption the investors consume all their wealth on a single pre-ordained day in the future. The Merton CAPM (otherwise known as the inter-temporal CAPM) generalised the Sharpe CAPM from a single period model of investment to a multi-period model of investment. Both Sharpe and Merton have been awarded the Nobel Prize in Economics for their contributions to finance theory.

129. By relaxing the single period assumption Merton showed that, in addition to a stock's covariance with the market at the end of the next period, an additional risk factor that also affects the return required by holders of the stock was the covariance of the asset's returns with the future profitability of reinvestment in the next period. The intuition behind this risk factor is relatively easy to understand. If an investor plans to consume their wealth over multiple periods then they will also care about the rate of return that they are able to achieve when they reinvest their earnings at the end of each period. Other things equal, they will naturally be attracted to stocks that have high pay-offs in times when re-investment opportunities are good and will shun stocks that do not have such high pay-offs in those times.

130. A numerical example can illustrate this point. Consider a two period model where the investor gets an opportunity to adjust (reinvest) their portfolio at the end of the first period. Imagine that there are two potential states of the world at the end of the first period and both have an equal 50% probability of being true: 1) the economy is in recession with little demand for capital and the average return on the market during the second period is only expected to be 5%; or 2) there is an economic boom, creating a demand for capital, and the average return on the market is expected to be 15%.

131. Now imagine two stocks, "*A*" and "*B*", that the investor is considering investing $100 in at the beginning of the first period. Stock *A* is expected to pay off $105 at the end of the first period in both recession and boom. Stock *B* is expected to payoff $40 in recession and $170 in boom. Note that both stocks have an identical expected (ie, probability weighted) pay-off of $105 at the end of period 1. That is, the expected return on both investments is 5% ($100 in and $105 out).

132. If we restrict ourselves, as Sharpe did, to only caring about our wealth at the end of the first period then stock *A* is clearly preferable to stock *B*. It has the same expected payoff but pays off more in a recession (when our well-diversified portfolio will not otherwise be very valuable and, if we are to consume our entire wealth, the marginal utility of an additional unit of consumption will be high) and less in a boom (when our consumption is already high). In the context of the single period Sharpe CAPM, stock *A* has the lower beta and so is more desirable than stock B (stock *A* has lower risk than *B*).

133. However, if we relax the one period assumption we find that stock *B* may actually be preferable to stock *A*. The reason why this is the case is relatively simple and intuitive. Stock *B* has the property that its return offers greater exposure to the higher returns available following a boom than does stock *A*. When an investor would otherwise cut back consumption to invest more given the improved investment opportunities, stock *B* provides its largest payoffs thereby allowing increased investment at a smaller cost to current consumption. That is, stock *B*

has higher 'good covariance' being the covariance with improvements in reinvestment opportunities.

134. The implication of this can be clearly seen by considering the expected pay-off over two periods from investing in stock *B* in the first period. If we invest in stock B in period one, we have $170 to reinvest when the return from investment is 15% and $40 to reinvest when returns from reinvesting are only 5%. This gives us expected wealth at the end of period two of $119 (being the sum of the probability weighted pay-offs for reinvestment in recession and boom (0.5*40*(1.05) plus 0.5*170*(1.15)). However, by the same process, the expected pay off from investing in stock *A* in period one can be determined to be $115 (=.5*105*(1.05) plus .5*105*(1.15)). Thus, over two periods, the expected return from investing in stock *B* during period one and in the market in period 2 is actually higher than the return from investing in stock A during period one and in the market in period 2.

135. Merton's insight was that having low Sharpe CAPM beta might actually raise a stock's risk relative to other stocks *if* it also meant that the asset had low positive correlation with improvements in investment opportunities that lead to increased investment in future periods (as is the case in the above scenario) compared to other stocks. Certainly, placing sole reliance on the Sharpe CAPM beta will lead to a downward biased estimate of relative risk in those circumstances.

136. Merton's CAPM provides a rigorous and intuitive explanation for why tests of the Sharpe CAPM have all shown that having sole regard to equity beta (i.e., having sole regard to only the <u>contemporaneous</u> covariance of stock and market returns) will underestimate required returns for stocks with low equity betas. Put simply, beta is not the only measure of risk relevant in more realistic CAPM models. Merton showed that, in the more realistic multi-period CAPM, assets with low betas (Sharpe CAPM covariance with the market) can nonetheless be high risk (if they have lower covariance than other assets with improvements in reinvestment opportunities: A low positive correlation with improvements in the opportunity set can mean a higher required return relative to stocks with a high positive correlation with improvements in the opportunity set.

137. Moreover, it is natural to expect, just as was the case in the above example, that firms with low β will tend to have low covariance with improvements in the reinvestment opportunity set (and *vice versa*). Put simply, if reinvestment opportunities are higher in booms than recessions such that an investor would optimally reduce consumption to invest more in response to the improved reinvestment opportunities then firms that payoff in recessions but not in booms will have relatively low beta risk but relatively high exposure to the risk associated with changes in the investment opportunity set. In other words, firms that fail to payoff in booms will be regarded as risky – because investment in these firms

mean your payoff is lower precisely when the returns from reinvestment are highest. Appendix A sets out the complex link between the required return on an asset and the covariance of returns on that asset with changes in future reinvestment opportunities.

138. The empirical success of the Fama-French book to market and size factors is currently the subject of much empirical and theoretical research investigating whether these factors are in fact proxies for a stock's exposure to reinvestment risk – see Berk, Green and Niak (1999) and Brennan, Wang and Xia (2004) .[19]

4.1.3. The Consumption CAPM – covariance with reinvestment matters

139. In a multiperiod setting investors will allocate their wealth between consumption and reinvestment for future periods. In a single period setting one's end-of-period wealth is one's end-of-period consumption and when the well-diversified market portfolio pays off more a respresentive consumer's marginal utility of consumption is lower. Stocks with higher betas tend to give their highest payoffs when the marginal utility of consumption is low and hence are not worth as much as a low beta stock with same end-of-period expected value. In fact, in the single period setting of the Sharpe CAPM, covariance with the market return is simply an alternate way of measuring This observation underlies the development of the Consumption CAPM (CCAPM) which has grown out of Merton's 1973 paper. In the Consumption CAPM investors attempt to maximise the present value of utility from all future periods of consumption. A result of this assumption it can be shown that the covariance that investors' care about is not the covariance of stock returns with the market (beta) but covariance of stock returns with contemporaneous consumption.

140. These will only amount to the same thing if investors' current consumption moves in proportion with wealth. However, there is good reason to believe that this will not be the case. In particular, as noted by Merton, changes in market discount rates will tend alter the trade off between current and future consumption. Higher discount rates imply higher expected returns which, other things equal, imply lower consumption today (as the cost of consumption today in terms of future consumption has increased).

141. The CCAPM can explain the observed weak relationship between beta and stock returns if:

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[19] Berk, J., R. Green, and V. Naik, 1999, "Optimal Investment, Growth Options and Security Returns," Journal of Finance 54, 1553-1608. Brennan, Michael J., Ashley W. Wang and Yihong Xia, 2004, "Estimation and Test of a Simple Model of Intertemporal Capital Asset Pricing," Journal of Finance 59, 1743-75.

i. there is less than perfect correlation between the return on the market and contemporaneous consumption;

ii. low beta stock tend to have exposure to events that cause the return on the market to be less than perfectly correlated with changes in consumption in a manner that means the return on such stock tend to have higher correlation with consumption than the market.

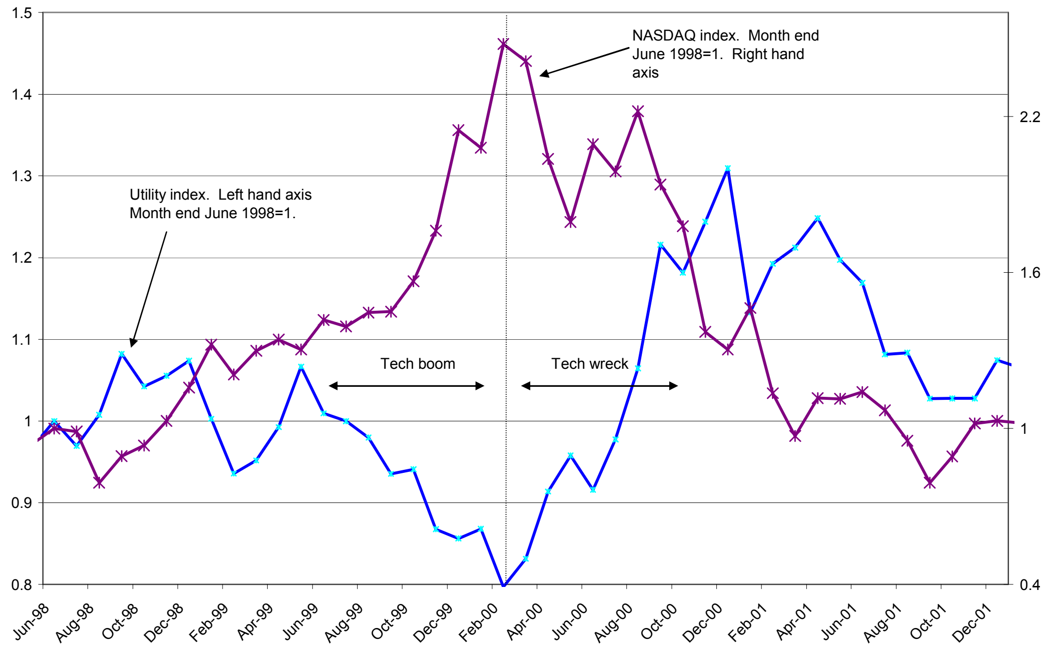
142. As shown in Appendix A, value stocks tend to have lower market betas than growth stocks yet on average earn higher returns. This is because, as is also seen in Appendix A, value stocks tend to covary more strongly with changes in consumption than do growth stock. They do so because of the difference in their sensitivity to changes in investment opportunities in the economy. The fact that low market beta value stocks have higher expected returns than high market beta growth stock implies that market beta is not the sole determinant of expected returns. This is consistent with theory, the US evidence, and the Australian evidence in Section 3, which demonstrated that low beta Australian stock earn more than would be predicted by the Sharpe CAPM.

4.2. Real world examples – the technology bubble and commodity boom?

143. In previous reports we have noted that the behaviour of utility stock prices during the technology bubble is consistent with the predictions of the Merton CAPM but not of the Sharpe CAPM.

144. Utility stocks moved counter-cyclically during the tech bubble – falling dramatically as the general stock market rose and rising dramatically as the general market fell. In fact, the turning points exactly match such that there can be little doubt that whatever was causing the rise in technology stocks was also causing the fall in utility stocks (and whatever was causing the subsequent fall in tech stocks was causing the subsequent rise in utility stocks). The figure below illustrates the relative performance of the US utilities (for which betas were estimated by the ESCV in its draft decision for Victorian Gas Distribution businesses) and the NASDAQ over the period of the tech boom and bubble.

Figure 8: Stock prices during the technology 'boom and bust'



145. Note that if the single period CAPM held then utility stocks should, other things equal, have stayed constant (or risen) during the boom because they would have offered better diversification opportunities relative to tech stocks (ie, their single period CAPM beta should have fallen during the boom).

146. Under the Sharpe CAPM if investors suddenly believe that one sector ('new economy' stock) has very high growth potential then the price of equity in that sector will go up but the value of other stocks will be largely unaffected. In fact, the value of other stocks should actually tend to increase because their weight in the market portfolio will fall and this will tend to reduce their equity beta (reduce their risk and therefore increase their price). Clearly, the Sharpe CAPM cannot easily explain the dramatic fall and rise of utility stocks during the tech bubble.

147. Under the Merton CAPM or the CCAPM, the rise of the 'new economy' sector can explain falling utility stocks. All that is required is that investors believe that 'new economy' stocks give them better exposure to higher reinvestment opportunities should the 'new economy' come to fruition. (Alternatively, in the context of the CCAPM, utilities will pay off less just when savings to take advantage of the new economy opportunities is at its highest.)

148. Consider our above example with stock A and B. Let stock A be the utility stock and stock B be a technology stock. Now instead of imagining investors perceive

the possibility of ‘boom’ or ‘recession’ imagine they perceive the possibility of ‘economy revolutionised by the internet’ or ‘economy not revolutionised by the internet’. If the expected reinvestment opportunities in the former exceed those in the latter then the Merton CAPM says investors will rationally treat utilities as higher risk (ie, holding utilities means you risk having less money to reinvest if the ‘new economy’ comes to fruition and reinvestment profits are high—less money that is, than if you had held new economy stocks). The Merton CAPM is consistent with what actually happened to utility prices.

149. It is relevant to note that the Australian economy and stock market is in the midst of another boom period related to commodity prices. Expressed in the popular language of the investment press one can conceive of a further two binary scenarios: i) the China wave keeps rolling ii) the China wave crashes. If investors believe that reinvestment opportunities will be higher in scenario i) then the Merton CAPM says they will treat stocks with little exposure to China as higher risk. To the extent that regulated utilities fall into this category (as seems reasonable to assume) this will raise the risk attached to investing in those utilities relative to the risk of investing in other stocks.

5. Recommendations

150. Currently, the National Electricity Rules (NER) prescribes the use of the Sharpe CAPM formula to calculate the required equity return:

$$R_f + \beta_e \cdot (R_m - R_f) \qquad \text{(i)}$$

where: R_f is the risk free rate (proxied in the NER in real terms by the prevailing yield on nominal Commonwealth Government bonds less an estimate of expected inflation);

β_e is the equity beta; and

$(R_m - R_f)$ is the expected market risk premium (MRP) being the expected return on the market less the risk free rate (the MRP is prescribed at a value of 6% in the NER).

151. The AER is running a consultation process on whether there is a need to amend the parameter values used in the above equation.

152. Our regression results clearly support findings from other markets and other time periods that:

i. firms with zero beta earn significantly more than the risk free rate; and

ii. that the sensitivity of equity returns is much lower than suggested by the Sharpe CAPM formula.

153. Specifically, our results suggest that, excluding the smallest stocks, higher betas tend to have no (or even a slightly negative) impact on expected returns. If we include the smallest stocks (and give an equal weighting to all stocks irrespective of size) we do find a strong and significant positive relationship between beta and stock returns. However, even in this circumstance we find that zero beta stocks earn well in excess of the risk free rate (7.26% in excess).

154. The most important conclusion to come out of this analysis is that it would be a mistake to simply take the best estimate of beta for utilities and insert this into the Sharpe CAPM formula. The Sharpe CAPM formula clearly does not explain how capital markets establish required returns. Even if the beta estimates used are perfectly accurate, inserting them into the wrong formula will give the wrong result.

155. More importantly, we know the direction of this error. If the beta estimates are below 1.0 then using these in the Sharpe CAPM will underestimate returns and *vice versa* if the beta estimates are above zero. If the regulator is restricted to have regard only to beta in setting regulated returns (eg, restricted to ignore size and/or other proxies for risk factors priced by the market) there are only two options for addressing the bias associated with the Sharpe CAPM formula:

i. Adopt a different formula where zero beta equity earn a positive excess return (ie, the Black CAPM formula rather than the Sharpe CAPM formula); or

ii. Set the value of β_e in the Sharpe CAPM formula in such a fashion as to remove the known bias.

156. Implemented consistently either approach will give the same result. Consequently, which of these options is adopted is a matter of form and not substance.

5.1. Adopting a different equation

157. The bias in the Sharpe CAPM formula used in the NER can be corrected by a simply adding an appropriate positive amount 'α' to the government bond rate. That is, increasing the risk free rate and removing the same amount from the MRP. This would adequately reflect the empirical fact that zero beta equity earns more than the Government bond rate. That is, the required equity return would be equal to:

$$R_f + \propto + \beta_e \cdot (R_m - R_f - \propto) \qquad \text{(ii)}$$

158. In this formulation, consistent with theory and empirics, equity with zero β_e earns on average more than the risk free rate and required returns increase more slowly as β_e increases. Based on our Australian empirical work the best estimate of "α" is approximately the same as $(R_m - R_f)$. That is, there does not appear to be any significant relation between β_e and equity returns in the Australian market. If we conservatively adopt a value for "α" that is 1% less than the value of β_e $(R_m - R_f)$ and the current NER value of 6% for $(R_m - R_f)$ is retained, this would imply a value for "α" of 5% and equation (ii) would be:

$$R_f + 5\% + \beta_e \cdot 1\%$$

159. Thus, if the best estimate of the value of β_e was 0.5 then the estimated cost of equity would be 5.5% in excess of the risk free rate (compared to 3.0% in excess of the risk free rate under equation (i)).

5.2. Adopting a different value of $\boldsymbol{\beta_e}$

160. Alternatively, if the Government bond rate (or similar) was retained as the risk free rate then the equity beta inserted into the Sharpe CAPM formula would need to be determined from the observed equity beta $(\beta_e{}^{\text{observed}})$ in the following manner:

$$\beta_e{}^{\text{equation (i)}} = \frac{\propto}{R_m - R_f} + \beta_e^{\text{observed}} \cdot (1 - \frac{\propto}{R_m - R_f})$$

161. This approach counteracts the bias by setting the equity beta above/below the observed equity beta when the observed equity beta is lower/higher than 1.0. This is mathematically equivalent to simply adopting equation (ii). For example, if we adopt the same parameter values (α = 5%, $(R_m - R_f)$ = 6% and $(\beta_e{}^{\text{observed}})$ = 0.5) then we get a value for beta to be used in equation (i) of 0.916* and this results in an estimated cost of equity 5.5% in excess of the risk free rate (as was the case above).

Appendix A. The Intertemporal CAPM (ICAPM)

A.1. Reinvestment risk in a multi-period setting

162. Consider a two-period setting. The first period runs from time $t - 1$ to time t and the second from time t to $t + 1$. A representative investor has wealth of W_t at the end of the first period. She divides her wealth at the end of the first period between time t consumption, C_t, and reinvestment of $[W_t - C_t]$.

163. In equilibrium, prices will adjust such that our representative investor holds the market portfolio. Reinvestment opportunities in the second period can be said to improve when, all else equal, either the real risk-free rate (r_f) increases, the market risk-premium ($E\{r_M\}$ - r_f) increases or the risk of the market (σ_M) decreases. When both the market risk-premium and variability increase, investment opportunities improve if the reward for risk-bearing has increased; i.e., if the slope of the capital market line, $\dfrac{E\{r_M\} - r_f}{\sigma_M}$, has increased. For ease of exposition, we assume that there is no uncertainty during the second period and hence all assets offer the risk-free rate during the second period. Thus the investment opportunity during the second period improves if the risk-free rate on offer during the second period is higher than was anticipated at the start of the first period.

164. We use the notation r_{ft} to denote the risk-free rate from time t to $t + 1$. Our representative investor's final consumption at $t + 1$, denoted by C_{t+1}, will equal the amount she invests at t plus the interest she earns thereon; that is $[W_t - C_t][1 + r_{ft}]$.

165. At time t our investor seeks to maximize the utility of her remaining lifetime consumption. Her time-additive, separable utility function takes the form

$$U(C_t) + \beta U(C_{t+1}),$$

where $U(C_\tau)$ is the utility from consuming C_τ at time τ, $U' > 0$ and $U'' < 0$, and β is a measures of her preference for current versus future consumption.

$$Max_{C_t} \quad U(C_t) + \beta U([W_t - C_t][1 + r_{ft}]).$$

At an optimum,

$$U'(C_t) - \beta U'([W_t - C_t][1 + r_{ft}])[1 + r_{ft}] = 0. \qquad \text{(A1)}$$

166. Applying the implicit function theorem to (A1) establishes the intuitive result that, holding the second period investment opportunity constant, the greater her wealth at time *t* the greater her consumption at time *t*.

$$\frac{\partial C_t}{\partial W_t} = -\frac{-\beta U''\left([W_t - C_t][1+r_{ft}]\right)[1+r_{ft}]^2}{U''(C_t) + \beta U''\left([W_t - C_t][1+r_{ft}]\right)[1+r_{ft}]^2} > 0 \,.$$

Holding constant the return available on reinvestment during the second period, the greater the realized return on the market over the first period the greater her consumption at time *t*, and hence the lower her marginal utility of consumption at time *t*.

167. Now consider two assets with equivalent expected values at the end of the first period, assets *A* and *B*. All else equal, the asset whose value covaries more strongly with the return on the market will tend to give its best payoffs when the marginal utility of additional consumption is lowest. This asset will be valued less highly at the start of the first period and we have the intuitive result that, all else equal, the asset with the higher market beta will be priced to offer the higher expected return.

168. But all else need not be equal. In this multi-period setting, the two assets may differ in their first period return covariance with changes in the second period investment opportunity. The implicit function theorem allows us to examine the relation between our investor's optimal end-of-first-period consumption and the return available from reinvestment during the second period holding constant her end-of-first-period wealth (i.e., for any given realization of the return on the market during the first period).

$$\begin{aligned}\frac{\partial C_t}{\partial r_{ft}} &= \frac{\beta U''\left([W_t - C_t][1+r_{ft}]\right)[W_t - C_t][1+r_{ft}] + \beta U'\left([W_t - C_t][1+r_{ft}]\right)}{U''(C_t) + \beta U''\left([W_t - C_t]\left[1+r_f(t, t+1)\right]\right)[1+r_{ft}]^2} \\ &= \frac{\beta U''(C_{t+1})C_{t+1} + \beta U'(C_{t+1})}{U''(C_t) + \beta U''(C_{t+1})[1+r_{ft}]^2} \\ &= \frac{\beta U'(C_{t+1})}{U''(C_t) + \beta U''(C_{t+1})[1+r_{ft}]^2}\left[1 - RRA(C_{t+1})\right],\end{aligned}$$

where $RRA(C) \equiv -\frac{U''(C)C}{U'(C)}$ is the investor's coefficient of relative risk aversion. If investor utility is such that $RRA(C) < 1$, $\frac{\partial C_t}{\partial r_{ft}} < 0$ and our representative investor consumes less and saves more when her investment opportunities improve—she prefers to cut-back consumption today in order to enjoy much more consumption tomorrow. But if investor utility is such that $RRA(C) > 1$, $\frac{\partial C_t}{\partial r_{ft}} > 0$ and our representative investor saves less and consumes more today when reinvestment opportunities improve.

169. Depending on the typical investors' coefficient of relative risk aversion, current consumption may either rise or fall in response to an improvement in future investment opportunities for any given level of wealth; i.e., an improvement in reinvestment opportunities could lead to either an increase or a decrease in the marginal utility of current consumption.

170. Consider again assets *A* and *B* with equivalent expected values at the end of the first period. Suppose that conditional on the market return during the first period, the end-of-first-period value of asset *A* covaries more positively with improvements in the second period investment opportunity set; i.e., asset *A* tends to worth more (less) at the end of the first period than asset *B* when the interest rate available on reinvestment during the second period is higher (lower) than initially anticipated. If a representative investor tends to save more and consume less when reinvestment opportunities improve, asset *A*'s end-of-first-period value will covary more positively with the end-of-period-one marginal utility of consumption than does asset *B*'s value. Asset *A* will give larger payoffs when payoffs are worth more. Asset *A* will then be worth more at the start of the first period and its required return over the first period will be lower. If though a representative investor tends to consume more and save less when reinvestment opportunities improve, asset *A* will be worth less than asset *B* at the start of the first period and *A*'s required return over the first period will be higher than that of *B*.

A.2. Do expected returns increase or decrease with an asset's covariance with improvements in reinvestment opportunities?

171. Campbell and Vuolteenaho (2004) examine whether an asset's covariance with improvements in reinvestment opportunities conditional on the realized return on the market affects investor's required return for holding the asset. To do so they examine an asset's return covariance with the return on the market, the familiar market beta, and the conditional covariance of its returns with improvements in the future investment opportunity set, where the conditioning is on the realized market return. Their study is motivated by their observation that:

> *"We know that value stocks outperform growth stocks, particularly among smaller stocks, and that this cannot be explained by the traditional static [i.e. Sharpe] CAPM. If the ICAPM is to explain this anomaly, then small*

growth stocks must have intertemporal hedging value that offsets their low returns; that is, their returns must be negatively correlated with innovations to investment opportunities."

172. We set out below a simplified variant of the Campbell Voulteenaho analysis. Investors in this economy consider the infinite series of future payoffs from assets. Assume that the future net cash flows (*NCF*) from the market follow a random walk; that is $E_t\left\{NCF_{t+1}\right\} = NCF_t$ for all *t*. Let ρ_t denote the discount rate at time *t* applicable to the perpetuity of expected future net cash flows in the economy as a whole. Let MV_t denote the market value of the market at time *t*.

173. Let upper case *R* denote simple returns. The one-plus rate of return from time *t* to *t* + 1 is $1+R_{t+1}$. Let lower case *r* denote continuously compounded returns. The continuously compounded return from *t* to time *t* + 1 r_{t+1}. The one-plus rate of return on the market is

$$
\begin{aligned}
1+R_{M,t+1} &\equiv \frac{MV_{t+1}+NCF_{t+1}}{MV_t} = \frac{\dfrac{E_{t+1}\left\{NCF_{t+2}\right\}}{\rho_{t+1}}+NCF_{t+1}}{\dfrac{E_t\left\{NCF_{t+1}\right\}}{\rho_t}} \\
&= \frac{\dfrac{NCF_{t+1}}{\rho_{t+1}}+NCF_{t+1}}{\dfrac{NCF_t}{\rho_t}} = \frac{NCF_{t+1}}{NCF_t}\rho_t\left(\frac{1+\rho_{t+1}}{\rho_{t+1}}\right) \\
&\approx \frac{NCF_{t+1}}{NCF_t}\left(\frac{\rho_t}{\rho_{t+1}}\right).
\end{aligned}
$$

$$
r_{M,t+1} = \ln\left(1+R_{M,t+1}\right) \approx \ln\left(\frac{NCF_{t+1}}{NCF_t}\right) - \ln\left(\frac{\rho_{t+1}}{\rho_t}\right).
$$

174. Let β_i denote the market beta of asset *i*.

$$\beta_i \approx \frac{\operatorname{cov}(r_{it}, r_{Mt})}{\sigma^2(r_{Mt})} = \frac{\operatorname{cov}\left(r_{it}, \ln\left(\frac{NCF_{t+1}}{NCF_t}\right) - \ln\left(\frac{\rho_{t+1}}{\rho_t}\right)\right)}{\sigma^2(r_{Mt})} = \frac{\operatorname{cov}\left(r_{it}, \ln\left(\frac{NCF_{t+1}}{NCF_t}\right)\right)}{\sigma^2(r_{Mt})} + \frac{\operatorname{cov}\left(r_{it}, -\ln\left(\frac{\rho_{t+1}}{\rho_t}\right)\right)}{\sigma^2(r_{Mt})}$$

$$= \beta_{i,CF} + \beta_{i,DR} ,$$

where $\beta_{i,CF} \equiv \frac{\operatorname{cov}\left(r_{it}, \ln\left(\frac{NCF_{t+1}}{NCF_t}\right)\right)}{\sigma^2(r_{Mt})}$ and $\beta_{i,DR} \equiv \frac{\operatorname{cov}\left(r_{it}, -\ln\left(\frac{\rho_{t+1}}{\rho_t}\right)\right)}{\sigma^2(r_{Mt})}$.

Thus we can express an asset's beta as the sum of its cash flow beta, its sensitivity to percent changes in aggregate net cash flows in the economy denoted by $\beta_{i,CF}$, and its discount rate beta, its sensitivity to percent changes in the future discount rate in the economy denoted by $\beta_{i,DR}$. Campbell and Vuolteenaho model changes in the investment opportunity set much more broadly than simply changes in the discount rate applicable to the market in a random walk world. But it is still the case that the market beta on an asset is just the sum of its cash flow beta and investment opportunity/discount rate betas.

175. The Intertemporal CAPM takes the form:

$$E\{r_i\} = r_f + \beta_i\left(E\{r_M\} - r_f\right)$$
$$+ \operatorname{cov}\left(r_i, -\%\text{ change in }\rho \mid r_M\right) \times \text{reward for bearing conditional reinvestment risk}$$

$$= r_f + \beta_i\left(E\{r_M\} - r_f\right)$$
$$+ \frac{\left[\beta_{i,DR}\sigma_M^2 - \beta_i \operatorname{cov}\left(r_M, -\%\text{ change in }\rho\right)\right]}{\operatorname{var}\left(-\%\text{ change in }\rho\right)} \times \text{reward for bearing conditional reinvestment risk}$$

$$= r_f + \beta_i\left(E\{r_M\} - r_f\right) + \beta_{i,DR}\left|r_m\right. \times \text{reward for bearing conditional reinvestment risk,}$$

where $\beta_{i,DR}\left|r_m\right. \equiv \frac{\left[\beta_{i,DR}\sigma_M^2 - \beta_i \operatorname{cov}\left(r_M, -\%\text{ change in }\rho\right)\right]}{\operatorname{var}\left(-\%\text{ change in }\rho\right)}$.

Combining the results in Tables 3 and 5 of Campbell and Vuolteenaho (2004) we obtain measures of β_i and $\beta_{i,DR}\left|r_m\right.$ for large and small value and growth firms.

	Growth Firms		Value Firms	
	β_i	$\beta_{i,DR} \vert r_m$	β_i	$\beta_{i,DR} \vert r_m$
Small Firms	1.72	0.42779	1.25	0.194082
Large Firms	1.03	0.264195	0.79	0.085019

176. Controlling for firm size, we see from the above table that value firms have smaller market betas than growth firms (1.25 versus 1.79 and 0.79 versus 1.03). Yet on average value firms earn higher returns than growth firms. This cannot be explained by the Sharpe CAPM. But we also see that growth firms have a higher covariance with diminutions in the investment opportunity set. The negative of the percentage increase in the discount rate applicable to future cash flow, $-\%$ change in ρ, is a measure of the diminution in investment opportunities.

177. Conditional on the market return, growth stock tend to payoff well relative to value stock when reinvestment opportunities decline; that is, growth stock tend to payoff well when consumption is lower than otherwise expected because traders in the market react to a decline in future investment opportunities by investing more and consuming less. Paragraph 7 of this appendix showed that if for the typical investor $RRA(C) > 1$, then a representative investor in the economy saves less and consumes more today when future reinvestment opportunities improve, and vice-versa.

Appendix B. Key findings in the empirical literature and bibliography

Author	Year	Study	Methodology	Conclusions
Black, Jensen, Scholes	1972	*The CAPM: Some Empirical Tests*	Regressions of portfolio returns on equity betas	• Sharpe CAPM rejected • Black CAPM accepted
Fama, MacBeth	1973	*Risk, Return, and Equilibrium: Empirical Tests*	Regressions of portfolio returns on equity betas, squared equity betas and idiosyncratic risk	• Sharpe CAPM rejected • Black CAPM accepted • Idiosyncratic risk does not earn a risk premium
Banz	1981	*The Relationship Between Return and Market Value of Common Stocks.*	Regressions of portfolio returns on equity betas and market value of firm's equity	• Stocks with small equity values (small stocks) have higher expected returns than stocks with large equity values (large stocks) even controlling for differences in equity betas. • Average returns are not explained by equity betas alone—there is a significant Size effect • Both Black and Sharpe CAPM models implicitly rejected. Merton CAPM models not implicitly rejected.
Keim	1983	*Size Related Anomalies and Stock Return Seasonality: Further*	Analysis of time series of abnormal returns (defined as	• The Size effect established by Banz is significant. There is also a significant "January effect"

		Empirical Evidence	differences between returns and Sharpe CAPM conditional expected returns) on Small Stocks and Large Stocks.	implying a seasonality in stock returns. • Both Black and Sharpe CAPM models implicitly rejected. Merton CAPM models not implicitly rejected.

Author	Year	Study	Methodology	Conclusions
Fama, French	1992 1995 1996 1998	*The Cross-Section of Expected Stock Returns* *Size and Book-to-Market Factors in Earnings and Returns* *Multifactor Explanations of Asset Pricing Anomalies* *Value Versus Growth: The International Evidence* *The Capital Asset*	Analysis of the relation between portfolio average returns and equity beta, market value of equity and book-to-market (B/M) ratios. Also analysis of the relation between portfolio average returns and portfolio return sensitivity to the return to the market (equity beta), the difference in returns on small versus large stocks (the SMB factor) and the difference in returns on high B/M stocks and low B/M stocks (the HML factor).	• Whether the model's problems reflect weaknesses in the theory or in its empirical implementation, the failure of the Sharpe CAPM in empirical tests implies that most applications of the model are invalid. • The relation between equity beta and average return is much flatter than the Sharpe CAPM predicts: The returns on low equity beta portfolios are higher, and the returns on high equity beta portfolios are lower than predicted. • Expected returns only appear to be related to equity betas when differences in equity betas are correlated with differences in the Size and B/M risks of the stocks examined. • Risks related to Size and B/M do explain differences in expected returns, irrespective of

.	2004 2006	*Pricing Model: Theory and Evidence* *The Value Premium and the CAPM*	Analysis of sub-periods and long time periods. Analysis of returns from many countries.	whether differences in Size and B/M are correlated with differences in equity betas. • Variation in equity betas that is unrelated to Size and B/M is unrelated to differences in the expected returns on stocks • Both Black and Sharpe CAPM models implicitly rejected. Focus on Merton CAPM models to explain the role of size and B/M as proxies for underlying beta risk.

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